Verifying Geometric Properties

Recall the following formulas:

1) Length of a line segment: $l = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

2) Midpoint of a line segment: $\text{midpt} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

3) Slope of a Line:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2-y_1}{x_2-x_1}$$

To classify a triangle, determine the length of all three sides. Classify by:

1) Scalene - all sides different lengths

2) Isosceles - two sides same length

3) Equilateral - all three sides same length

To determine if a triangle is right-angled:

Method1 Find the slope of each side of the triangle. If the slopes of 2 sides are negative reciprocals, then the Δ is right-angled.

Method2 Using the lengths of all three sides, determine if the pythagorean theorem holds true. ie. Does $a^2+b^2=c^2$? (where c is the longest side)

Recall: a quadrilateral is

To classify a quadrilateral, determine the length and slope of each side. Classify by:

quadrilateral - 4 sided polygon

1) Parallelogram – quadrilateral with opposite sides equal in length and having same slope

2) Rectangle – quadrilateral with opposite sides equal in length and having same slope, and adjacent sides are perpendicular

3) Square – quadrilateral with opposite sides parallel, all sides same length and adjacent sides perpendicular

4) Rhombus –

quadrilateral with opposite sides parallel (same slope) and all 4 sides have same length.

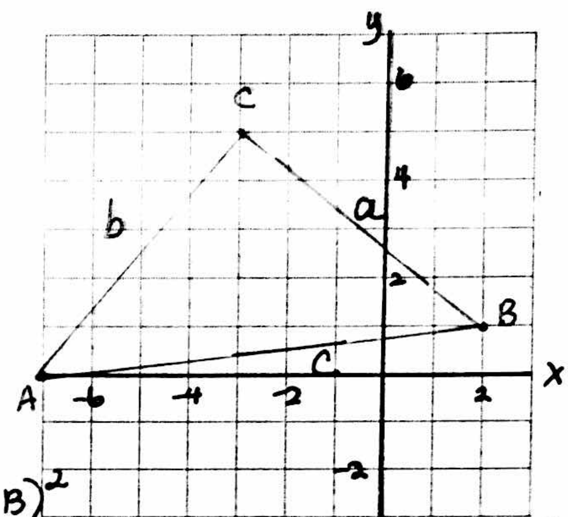
Eg. 1 A triangle has vertices A (-7,0), B (2,1), and C (-3,5). Classify the type of triangle and verify that it is right-angled.

$l_{AB} = \sqrt{(2+7)^2 + (1-0)^2}$
$= \sqrt{(9)^2 + (1)^2}$
$= \sqrt{82}$

$l_{BC} = \sqrt{(-3-2)^2 + (5-1)^2}$
$= \sqrt{(-5)^2 + (4)^2}$
$= \sqrt{25+16}$
$= \sqrt{41}$

$l_{AC} = \sqrt{(-3+7)^2 + (5-0)^2}$
$= \sqrt{(4)^2 + (5)^2}$
$= \sqrt{16+25}$
$= \sqrt{41}$

$\because l_{AC} = l_{BC} \neq l_{AB}$
then $\triangle ABC$ is isosceles.



Check the pyth. thm:

$LS = (AC)^2 + (BC)^2$
$= b^2 + a^2$
$= (\sqrt{41})^2 + (\sqrt{41})^2$
$= 41 + 41 = 82$

$RS = (AB)^2$
$= c^2$
$= (\sqrt{82})^2$
$= 82$

$\because LS = RS$, then $a^2+b^2=c^2$ so the triangle is right-angled.

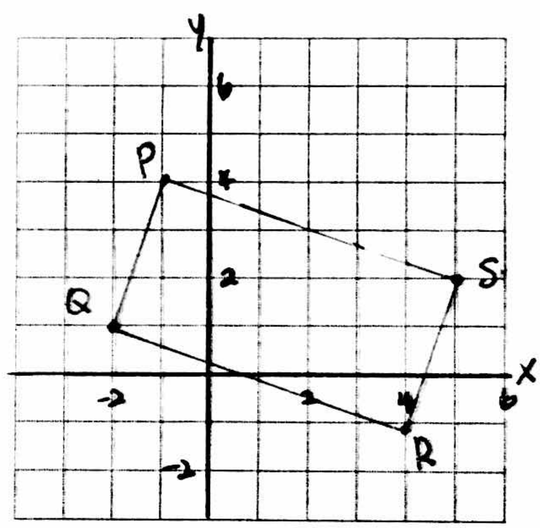
Eg. 2 Verify that the quadrilateral with vertices P (-1,4), Q (-2,1), R (4, -1), and S (5,2) is a rectangle.

$m_{RS} = \frac{2+1}{5-4}$
$= 3$

$m_{QP} = \frac{1-4}{-2+1}$
$= \frac{-3}{-1} = 3$

$m_{PS} = \frac{2-4}{5+1}$
$= \frac{-2}{6} = \frac{-1}{3}$

$m_{QR} = \frac{-1-1}{4+2}$
$= \frac{-2}{6} = \frac{-1}{3}$



$\because$ Opposite sides have the same slope and adjacent sides have slopes that are negative reciprocals then PQRS is a rectangle

* note: It follows from above that opposite sides would have to also be the same length.