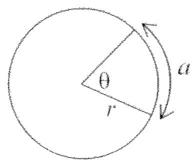
Trigonometry Review

1) Radian Measure

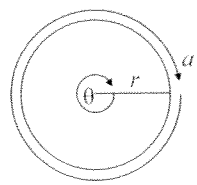
Angles can be measured in degrees or radians, but radians are generally used in calculus.

Given:



$$\boxed{\theta = \frac{a}{r} = \frac{\text{arc length}}{\text{radius}}}$$

In one revolution:



$\theta = \frac{a}{r}$

$\theta = \frac{2\pi r}{r}$ → circumference

$\theta = 2\pi$ radians $= 360°$

$\boxed{\pi \text{ radians} = 180°}$ or $\frac{\pi}{180}$ rad $= 1°$

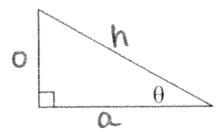
Eg. Convert the following angles to radian measure:

a) $36° = 36 \times \frac{\pi}{180} = \frac{\pi}{5}$ rad

b) $-135° = -135 \times \frac{\pi}{180} = \frac{-3\pi}{4}$ rad

2) Definitions of Trig Functions

In any right-angled triangle:



$\sin\theta = \frac{o}{h}$

$\cos\theta = \frac{a}{h}$

$\tan\theta = \frac{o}{a}$

$\csc\theta = \frac{1}{\sin\theta} = \frac{h}{o}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{h}{a}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{a}{o}$

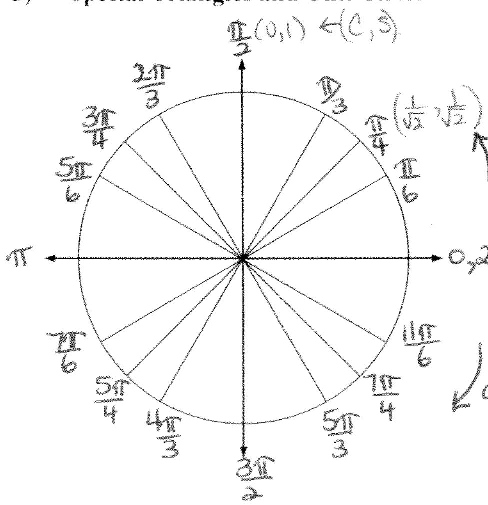
Also: $\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\cot\theta = \frac{\cos\theta}{\tan\theta}$

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$ ($\div$ by $\cos^2\theta$)

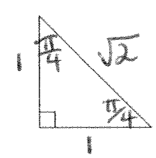
$1 + \cot^2\theta = \csc^2\theta$ ($\div$ by $\sin^2\theta$)

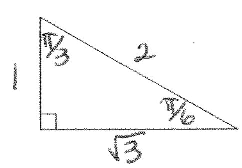
3) Special Triangles and Unit Circle



↑ + angles

↓ − angles





$\frac{\pi}{4} = 45°$

$\frac{\pi}{6} = 30°$

$\frac{\pi}{3} = 60°$

Note: $\sin(-\theta) = -\sin\theta$
$\tan(-\theta) = -\tan\theta$
$\cos(-\theta) = \cos\theta$

CAST Rule:

(2nd Quad) S	A (1st Quad)
sin + cos − tan −	sin + cos + tan +
sin − cos − tan +	sin − cos + tan −
(3rd Quad) T	C (4th Quad)

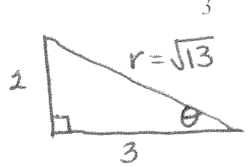
Eg. Evaluate the following:

a) $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

b) $\cos\left(\frac{3\pi}{4}\right) = -\frac{1}{\sqrt{2}}$

c) $\sin\left(\frac{-2\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

Eg. Given $\tan\theta = \frac{2}{3}$ and $\pi \le \theta \le \frac{3\pi}{2}$, determine the values of $\sin\theta$ and $\cos\theta$.



$r = \sqrt{2^2 + 3^2}$
$= \sqrt{4+9}$
$= \sqrt{13}$

In 3rd Quad, both $\sin\theta$ and $\cos\theta$ are neg.

$\sin\theta = -\frac{2}{\sqrt{13}}$ $\cos\theta = \frac{-3}{\sqrt{13}}$

4) Compound Angle Identities

Addition/Subtraction Identities:

$\cos(A+B)=\cos A\cos B-\sin A\sin B$

$\cos(A-B)=\cos A\cos B+\sin A\sin B$

$\sin(A+B)=\sin A\cos B+\cos A\sin B$

$\sin(A-B)=\sin A\cos B-\cos A\sin B$

$\tan(A+B)=\dfrac{\tan A+\tan B}{1-\tan A\tan B}$

$\tan(A-B)=\dfrac{\tan A-\tan B}{1+\tan A\tan B}$

Double-Angle Identities:

$\sin 2A=2\sin A\cos A$

$\cos 2A=\cos^2 A-\sin^2 A$

$=2\cos^2 A-1$

$=1-2\sin^2 A$

$\tan 2A=\dfrac{2\tan A}{1-\tan^2 A}$

Eg. Show that $\cos\left(\frac{\pi}{2}-A\right)=\sin A$.

*Note also: $\sin\left(\frac{\pi}{2}-A\right)=\cos A$

$$\cos\left(\frac{\pi}{2}-A\right)=\cos\frac{\pi}{2}\cos A+\sin\frac{\pi}{2}\sin A$$
$$=0\cdot\cos A+1\cdot\sin A$$
$$=0+\sin A$$
$$=\sin A$$

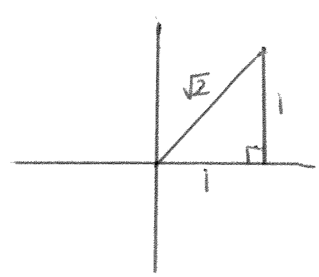
Eg. Prove the following trig identity: $\dfrac{\sin(x+y)}{\sin x\cos y}=1+\cot x\tan y$

$$LS=\frac{\sin(x+y)}{\sin x\cos y}$$
$$=\frac{\sin x\cos y+\cos x\sin y}{\sin x\cos y}$$
$$=1+\frac{\cos x\sin y}{\sin x\cos y}$$
$$=1+\cot x\tan y=RS.$$

Eg. Solve the following equation for x, where $0\le x\le 2\pi$.

$\cos 2x=\cancel{1+2\sin x}$ $2\sin^2 x-1$

$$\cos 2x=2\sin^2 x-1$$
$$\cos^2 x-\sin^2 x=2\sin^2 x-1$$
$$(1-\sin^2 x)-\sin^2 x=2\sin^2 x-1$$
$$1-2\sin^2 x=2\sin^2 x-1$$
$$2=4\sin^2 x$$
$$\frac{1}{2}=\sin^2 x$$
$$\pm\frac{1}{\sqrt{2}}=\sin x$$



$$x=\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4}$$