DERIVATIVES OF SIN X AND COS X

1. **Prove that** $\frac{d}{dx}\sin x = \cos x$**.**

Let $y = \sin x$

From first principles,

$$y' = \lim_{h\to 0}\left[\frac{\sin(x+h)-\sin(x)}{h}\right]$$

$$= \lim_{h\to 0}\left[\frac{(\sin x\cdot\cos h+\sin h\cdot\cos x)-\sin x}{h}\right]$$

$$= \lim_{h\to 0}\left[\frac{\sin x(\cos h-1)}{h}+\frac{\sin h\cdot\cos x}{h}\right]$$

$$= \lim_{h\to 0}\left[\sin x\cdot\frac{\cos h-1}{h}\right]+\lim_{h\to 0}\left[\frac{\sin h}{h}\cdot\cos x\right]$$

$$= \sin x\cdot\lim_{h\to 0}\left[\frac{\cos h-1}{h}\right]+\cos x\cdot\lim_{h\to 0}\left[\frac{\sin h}{h}\right]$$

$$= \sin x\cdot(0)+\cos x\cdot(1)$$

$$\therefore y' = \cos x$$

2. **Prove that** $\frac{d}{dx}\cos x = -\sin x$**.**

Let $y = \cos x = \sin(\frac{\pi}{2}-x)$

Using the results from above,

$$y' = \cos(\tfrac{\pi}{2}-x)\cdot(-1)$$

$$= \sin x\cdot(-1)$$

$$= -\sin x$$

Derivative of the TAN function:

We know $\tan x = \dfrac{\sin x}{\cos x}$

Using the Quotient Rule: $\left[\dfrac{f(x)}{g(x)}\right]' = \dfrac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$

Let $f(x) = \sin x$ & $g(x) = \cos x$

$\therefore f'(x) = \cos x$ $\therefore g'(x) = -\sin x$

$$\frac{d}{dx}(\tan x) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$

$$= \frac{\cos x \cos x - \sin x(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x}$$

$$= \sec^2 x$$

$$\boxed{\frac{d}{dx}(\tan x) = \sec^2 x}$$