Derivatives of Trig Functions

Given: $\frac{d}{dx}\sin x = \cos x$ $\quad$ $\frac{d}{dx}\cos x = -\sin x$ (see proofs)

Eg. Differentiate

① $y = \sin 3x$

$\frac{dy}{dx} = \cos 3x\,(3)$

$= 3\cos 3x$

② $y = -2\cos x^2$

$\frac{dy}{dx} = -2(-\sin x^2)(2x)$

$= 4x \sin x^2$

③ $y = x^2 \cdot \sin x$

$\frac{dy}{dx} = 2x\sin x + x^2(\cos x)$

$= x(2\sin x + x\cos x)$

④ $y = \tan x$

$y = \frac{\sin x}{\cos x}$

$\frac{dy}{dx} = \frac{\cos x(\cos x) - \sin x(-\sin x)}{\cos^2 x}$

$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$

$= \frac{1}{\cos^2 x}$

$= \sec^2 x$

Additional Trig Derivatives

$\frac{d}{dx}\tan x = \sec^2 x$

$\frac{d}{dx}\cot x = -\csc^2 x$

$\frac{d}{dx}\sec x = \sec x \tan x$

$\frac{d}{dx}\csc x = -\csc x \cot x$

⑤ $y = \csc\sqrt{x}$

$\frac{dy}{dx} = -\csc\sqrt{x}\cot\sqrt{x}\left(\frac{1}{2}x^{-1/2}\right)$

$= \frac{-1}{2\sqrt{x}}\csc\sqrt{x}\cot\sqrt{x}$

⑥ $y = \cot(2x+1)^4$

$$\frac{dy}{dx} = -\csc^2(2x+1)^4 \left[4(2x+1)^3(2)\right]$$
$$= -8(2x+1)^3 \csc^2(2x+1)^4$$

⑦ $y = \cot^4(2x+1)$

$$\frac{dy}{dx} = 4[\cot(2x+1)]^3 [-\csc^2(2x+1)] (2)$$
$$= -8\cot^3(2x+1)\csc^2(2x+1)$$

⑧ $1 - \cos(x+y) = \sec y$

$$-(-\sin(x+y))\left[1 + \frac{dy}{dx}\right] = \sec y \tan y \cdot \frac{dy}{dx}$$

$$\sin(x+y) + \frac{dy}{dx}\sin(x+y) = \sec y \tan y \cdot \frac{dy}{dx}$$

$$\frac{dy}{dx}\left[\sin(x+y) - \sec y \tan y\right] = -\sin(x+y)$$

$$\frac{dy}{dx} = \frac{-\sin(x+y)}{\sin(x+y) - \sec y \tan y}$$