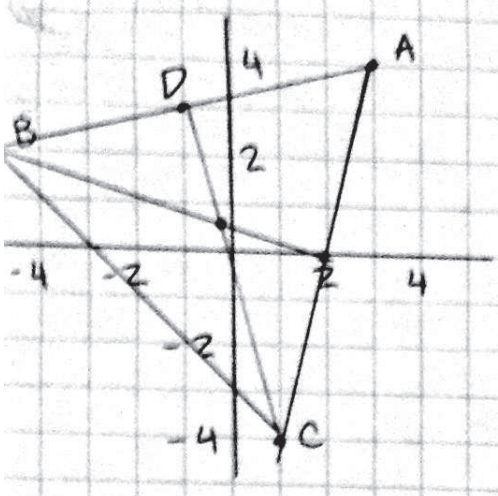
From example 1: median CD - $7x + 2y + 1 = 0$ (right bisector of AB)



A(3,4) B(-5,2) C(1,-4)
D(-1,3) E(2,0)

1) Find midpt AC

$$\text{midpt}_{AC} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = E$$

$$= \left(\frac{3+1}{2}, \frac{4-4}{2}\right)$$

$$= \left(\frac{4}{2}, \frac{0}{2}\right)$$

$$E = (2, 0)$$

2) Find slope of BE

$$m_{BE} = \frac{\Delta y}{\Delta x}$$

$$= \frac{0-2}{2+5}$$

$$= \frac{-2}{7}$$

3) equation

$$m = \frac{y-y_1}{x-x_1}$$

$$-\frac{2}{7} = \frac{y-0}{x-2}$$

$$-2x+4 = 7y-0$$

$$-2x-7y+4 = 0$$

$$2x+7y-4 = 0$$

4) solve

$7x + 2y + 1 = 0$ ①

$2x + 7y - 4 = 0$ ②

① × 2: $14x + 4y + 2 = 0$

② × 7: $14x + 49y - 28 = 0$

$-45y + 30 = 0$

$$\frac{30}{45} = \frac{45y}{45}$$

$$\frac{6}{9} = y$$

$$y = \frac{2}{3}$$

sub $y = \frac{2}{3}$ into ③

$2x + 7\left(\frac{2}{3}\right) - 4 = 0$

$6x + 14 - 12 = 0$

$6x = -14 + 12$

$6x = -2$

$x = -\frac{2}{6}$

$x = -\frac{1}{3}$

∴ the centroid of $\triangle ABC$ is $\left(-\frac{1}{3}, \frac{2}{3}\right)$

Circumcentre

From yesterday's note, the equation of the right bisector GH is $4x+y+1=0$.

$\text{midpt}_{BC} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
$= \left(\frac{-5+1}{2}, \frac{2-4}{2}\right)$
$= (-2, -1)$

$m_{BC} = \frac{y_2-y_1}{x_2-x_1}$
$= \frac{-4-2}{1+5}$
$= -1$

Since the slope of the right bisector DA is $\perp$ to BC, the slope is 1.

Equation: $m_{BC} = 1$

$D = (-2, -1)$

$m = \frac{y-y_1}{x-x_1}$

$\frac{1}{1} = \frac{y+1}{x+2}$

$x+2 = y+1$

$x - y + 1 = 0$

$4x+y+1=0$ ①

$x - y + 1 = 0$ ②

From ①: $x = y-1$ ③

Sub ③ in ①: $4(y-1)+y+1=0$

$4y - 4 + y + 1 = 0$

$5y - 3 = 0$

$5y = 3$

$y = \frac{3}{5}$

Sub $y = \frac{3}{5}$ in ②: $x - \frac{3}{5} + 1 = 0$

$5x - 3 + 5 = 0$

$5x = -2$

$x = \frac{-2}{5}$

$\therefore$ The circumcentre is $\left(\frac{-2}{5}, \frac{3}{5}\right)$

OrthoCentre

02/10/2014

* From yesterday's note, the equation of the altitude was $0 = 4x + y$

Altitude line BC

$$\text{Slope} = m_{BC} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{4-2}{1+5}$$
$$= \frac{-6}{6}$$
$$= -1$$

$m_{AC} \perp m_{BC} = 1$

$$\text{Equation} = \frac{1}{1} = \frac{y-4}{x-3}$$
$$x - 3 = y - 4$$
$$x - y + 1 = 0$$

Orthocentre

$x - y + 1 = 0$
$4x + y = 0$
(add)
$5x + 1 = 0$
$5x = -1$
$x = \frac{-1}{5}$

$x - y + 1 = 0$ ①
$4x + y = 0$

① × 4: $4x - 4y + 4 = 0$

(Subtract)
$-5y = -4$
$y = \frac{-4}{-5}$
$y = \frac{4}{5}$

$\therefore$ Orthocentre $= x: \frac{-1}{5} \quad y: \frac{4}{5}$