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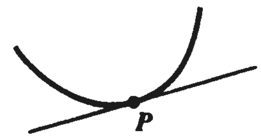
MCV 4UI – *The Slope of a Tangent*

RECALL: The slope of a linear function is the "rate of change of y with respect to x", and is given by:

$$m=\frac{\Delta y}{\Delta x}=\frac{y_2-y_1}{x_2-x_1}$$ (given 2 points (x_1,y_1) and (x_2,y_2))

Tangent:

- a line that touches a curve at one point and best approximates the slope of the curve at that point



Secant:

- a line that cuts through a curve at two points

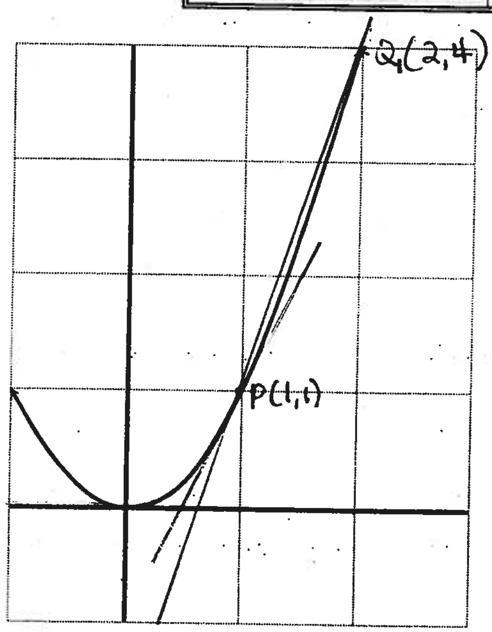
What if we are asked to find the slope of the tangent line to the curve $\boxed{y=x^2}$ at the point $P(1, 1)$?

The Problem: To find slope, you need ***two*** points, but we only have ***one***!

The Solution: Use another point on the curve that is very close to P and approximate the slope

Point P	Point Q	Slope of Secant PQ
(1, 1)	(2, 4)	$m=\frac{4-1}{2-1}=3$
(1, 1)	(1.5, 2.25)	$m=\frac{2.25-1}{1.5-1}=2.5$
(1, 1)	(1.25, 1.5625)	$m=\frac{1.5625-1}{1.25-1}=2.25$
(1, 1)	(1.1, 1.21)	$m=\frac{1.21-1}{1.1-1}=2.1$
(1, 1)	(1.01, 1.0201)	$m=\frac{1.0201-1}{1.01-1}=2.01$

↓ 2



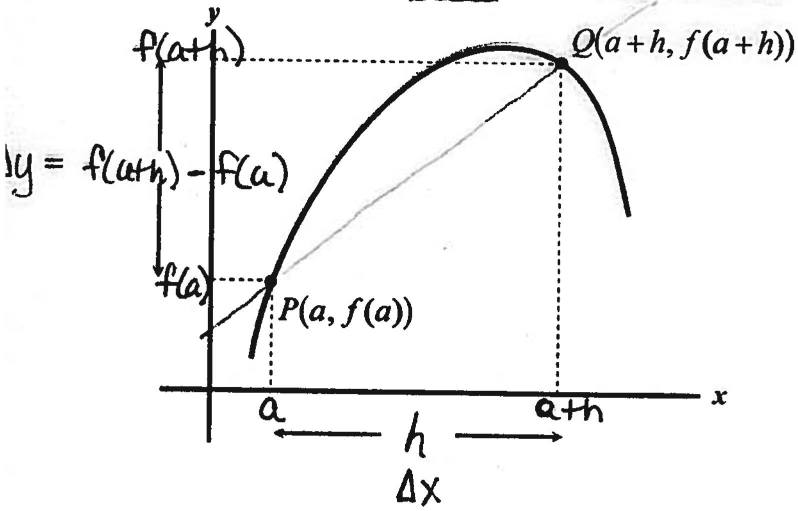
Notice: **a)** As Q gets closer to P, the slope of PQ gets closer to the value of 2.

b) As Q gets closer to P, the secant PQ tends to look more like a tangent at P.

c) The same thing will happen if we approach P from the *left* side of the curve!

Therefore, the slope of the tangent to a curve at a point P is the ***limiting slope of the secant PQ*** as the point Q "slides" along the curve towards P. We can generalize the method above to find the slope of the tangent to the graph of any function $y = f(x)$.

Let $P(a, f(a))$ be a fixed point on the graph of $\boxed{y = f(x)}$. If we pick another point, Q, which is a horizontal distance of h units from P, then we have.



Therefore, the slope of the secant PQ is:

$$m_{PQ} = \frac{\Delta y}{\Delta x}$$

$$m_{PQ} = \frac{f(a+h) - f(a)}{h}$$

Now, as Q approaches P along the curve, $h \Rightarrow 0$, and the secant becomes a tangent at P.

Therefore, the slope of the tangent at P is given by:

$$m_{\tan} = \lim_{h \to 0}(\text{slope of secant } PQ) = \boxed{\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}}$$

Using the definition of slope of a tangent, we can now prove that the slope of the tangent line to the curve $y = x^2$ at the point $P(1, 1)$ is actually 2.

Eg. a) Show the slope of the tangent to $y = x^2$ at the point $P(1, 1)$ is 2.

$a = 1$

$$m_{\tan} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

At pt $(1,1) \Rightarrow a = 1$

$$m_{\tan} = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0} \frac{[1+2h+h^2] - [1]}{h}$$

$$= \lim_{h \to 0} \frac{2h+h^2}{h}$$

$f(x) = x^2$

$f(1) = (1)^2 = 1$

$f(1+h) = (1+h)^2$

$= 1 + 2h + h^2$

$$= \lim_{h \to 0} \frac{\cancel{h}(2+h)}{\cancel{h}}$$

$= 2 + 0 = 2$ ← slope of tangent to $y = x^2$ at pt. $(1,1)$

b) Find the equation of the tangent line at the point $P(1, 1)$.

$m = 2$
pt $(1,1)$

$y = mx + b$
$(1) = 2(1) + b$
$1 - 2 = b$
$-1 = b$

$\therefore y = 2x - 1$

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