MCV 4U1 Feb 24/12

The Quotient Rule

When a function is expressed as the quotient of two other functions, ie. $f(x) = \frac{g(x)}{k(x)}$

then we can determine the derivative using the Quotient Rule: $\boxed{f'(x) = \frac{g'(x) \cdot k(x) - g(x) \cdot k'(x)}{[k(x)]^2}}$

*we will examine the proof next week

Eg1 Differentiate and simplify.

(a) $f(x) = \frac{2x+1}{x-2}$ ← $g(x)$ ← $k(x)$

$$f'(x) = \frac{\overbrace{(2)}^{g'(x)}\overbrace{(x-2)}^{k(x)} - \overbrace{(2x+1)}^{g(x)}\overbrace{(1)}^{k'(x)}}{\underbrace{(x-2)^2}_{[k'(x)]^2}}$$

$$= \frac{2x-4-2x-1}{(x-2)^2}$$

$$= \frac{-5}{(x-2)^2}$$

*When simplifying, expand and simplify the numerator, but leave the denominator in factored form.

(b) $f(x) = \frac{2x^3}{\sqrt{x}-4} = \frac{2x^3}{x^{1/2}-4}$

$$f'(x) = \frac{(6x^2)(x^{1/2}-4) - (2x^3)(\frac{1}{2}x^{-1/2})}{(x^{1/2}-4)^2}$$

$$= \frac{6x^{5/2} - 24x^2 - x^{5/2}}{(x^{1/2}-4)^2} = \frac{5x^{5/2} - 24x^2}{(\sqrt{x}-4)^2}$$

Eg2 Find the equation of the tangent to the curve $y=\frac{x^2-1}{2+x^3}$ at the point $(1,0)$.

$$\frac{dy}{dx}=\frac{(2x)(2+x^3)-(x^2-1)(3x^2)}{(2+x^3)^2}$$

$$=\frac{4x+2x^4-3x^4+3x^2}{(2+x^3)^2}$$

$$=\frac{4x+3x^2-x^4}{(2+x^3)^2}$$

$$m_{tan}=\left.\frac{dy}{dx}\right|_{x=1}$$

means sub. $x=1$ into $\frac{dy}{dx}$

$$=\frac{4(1)+3(1)^2-(1)^4}{(2+1^3)^2}$$

$$=\frac{6}{9}=\frac{2}{3}$$

$\therefore$ Equation of the tangent at $(1,0)$ with $m=\frac{2}{3}$:

$$\frac{2}{3}=\frac{y-0}{x-1}$$

$$2x-2=3y$$
$$2x-3y-2=0$$

Eg3 Find the point on the graph of $f(x)=\frac{2x+8}{\sqrt{x}}$ where the tangent is horizontal.

$$f'(x)=\frac{(2)(\sqrt{x})-(2x+8)(\frac{1}{2}x^{-1/2})}{(\sqrt{x})^2}$$

$$=\frac{2x^{1/2}-x^{1/2}-4x^{-1/2}}{x}$$

We usually simplify the denominator if it is just a single term.

$$=\frac{x^{1/2}-4x^{-1/2}}{x}$$

$$f'(x) = x^{-1/2} - 4x^{-3/2}$$
$$= \frac{1}{\sqrt{x}} - \frac{4}{(\sqrt{x})^3}$$
$$= \frac{x-4}{(\sqrt{x})^3}$$

Simplify the numerator so there are no negative exponents.

For a horizontal tangent, $m_{tan} = 0 \Rightarrow f'(x) = 0$

Solve: $f'(x) = \frac{x-4}{(\sqrt{x})^3} = 0$

$x - 4 = 0$
$x = 4$

Find y-value:
$y = \frac{2(4)+8}{\sqrt{4}}$
$= \frac{16}{\sqrt{2}} = 8$

$\therefore$ The point on the graph where the tangent is horizontal is $(4, 8)$.

HW: pg 150 #4 odds, 5ab, 7, 9a