The Derivative Function

Recall: Given a function $f(x)$ then:

$$m_{tan} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$ ← slope of the tangent to $y=f(x)$ at $x=a$

Now: Given a function $y=f(x)$ define:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$ ← the derivative of $y=f(x)$ with respect to x

read as "f prime at x"

Other notations used to represent the derivative:

$$f'(x),\ \frac{dy}{dx},\ y'$$

$\frac{dy}{dx}$: read as "dee y by dee x"

y' → read as "y prime"

There are two interpretations of the derivative:

1) slope of the tangent to a curve
2) the "instantaneous" rate of change of one variable with respect to another

eg. velocity is the rate of change of position w.r.t. time
w.r.t. = with respect to

The process of taking the derivative of a function is called differentiation.

The derivative of a function, $f(x)$, is also a function of x. ie. $f'(x)$.

Eg1 a) Find the derivative of $f(x)=x^2$ by $\underbrace{\text{first principles}}_{\text{using the limit definition}}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$* \ f(x+h) = (x+h)^2$
$= x^2+2xh+h^2$

$$= \lim_{h\to 0} \frac{[\cancel{x^2}+2xh+h^2]-[\cancel{x^2}]}{h}$$

$$= \lim_{h\to 0} \frac{h(2x+h)}{h}$$

$$= 2x+(0)$$

$$f'(x) = 2x$$

b) Evaluate: (i) $f'(3)$ ← sub $x=3$ into $f'(x)$

$= 2(3)$

represents the slope of the tangent to $f(x)=x^2$ at $x=3$ → $=6$

(ii) $f'(-1)$

$= 2(-1)$

$= -2$

Eg2 Find the derivative of $y=\frac{1}{1+x}$ by first principles

$$y' = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{1+(x+h)}-\frac{1}{1+x}}{h}$$

$$= \lim_{h\to 0} \frac{(\cancel{1}+\cancel{x})-(\cancel{1}+\cancel{x}+h)}{(1+x)(1+x+h)} \cdot \frac{1}{h}$$

$$= \lim_{h\to 0} \frac{\overset{-1}{\cancel{-h}}}{(1+x)(1+x+h)} \cdot \frac{1}{\cancel{h}}$$

Eg2 cont'd

$$y' = \frac{-1}{(1+x)(1+x+0)}$$

$$= \frac{-1}{(1+x)^2}$$

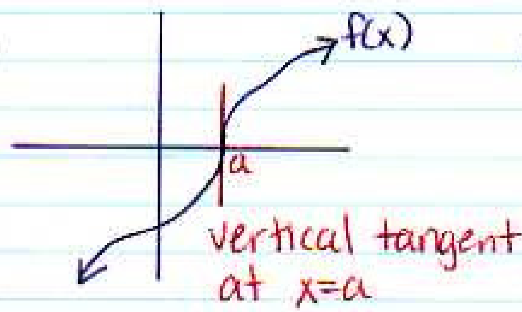
Differentiability

A function is said to be differentiable at a point $x=a$ if $f'(a)$ exists.

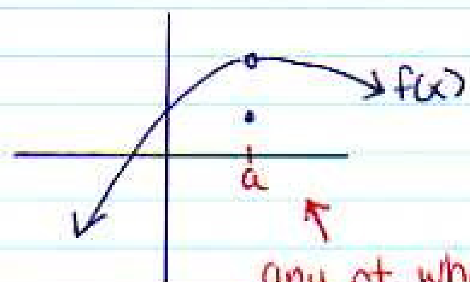
ie. if $\lim\limits_{h\to 0} \frac{f(x+h)-f(x)}{h}$ exists at $x=a$

Examples of functions which are not differentiable at $x=a$:

(a)

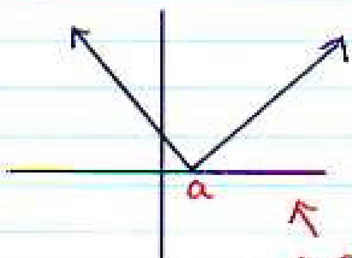


(b)

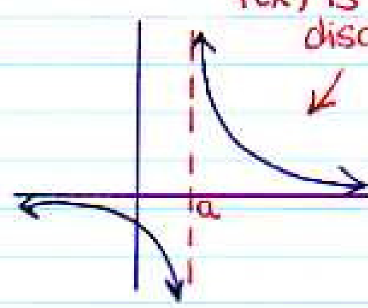


any pt. where f(x) is discontinuous

(c)

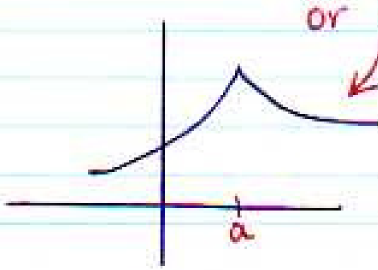


(d)



a sharp pt. or cusp

(e)



HW: pg 130 #1, 4-7, 12
Worksheet "Differentiability"