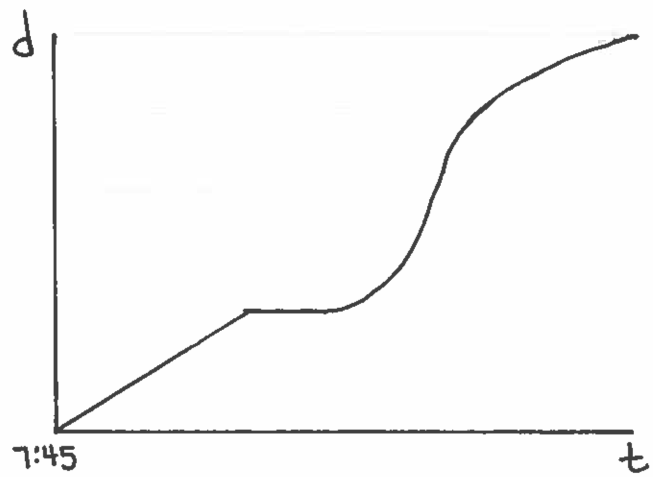
Recognizing Rates of Change

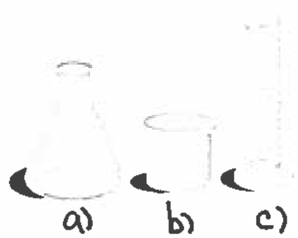
We are often interested in how quickly the dependent variable of a function changes when there is a change in the independent variable. This concept is called the **rate of change**.
Some common rates of change that you are already familiar with are:

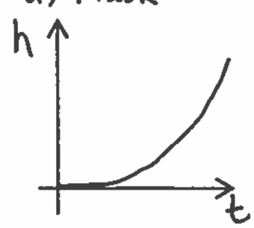
- Slope (rate of change of y with respect to x)
- Speed (rate of change of distance travelled with respect to time)
- Fuel consumption rate (rate of amount of fuel used per 100 km driven)

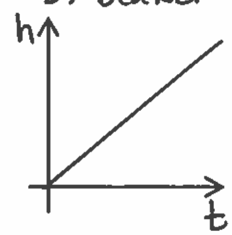
Eg. 1 Draw a graph to represent Jordan's distance (d) from home over time (t).
Jordan left home at 7:45 am and started walking slowly to school. He walked at a constant rate for 5 minutes then he stopped at his friend's house and waited 2 minutes for his friend to join him. Together they started walking towards school, increasing their speed for the next 3 minutes as they thought they would be late. Realizing they still had plenty of time, they gradually slowed their pace until they reached the school at 8:00 am.

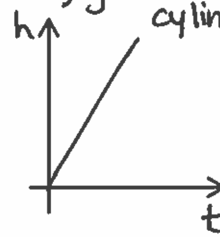


Eg. 2 A flask, a beaker and a graduated cylinder are being filled with water. The rate at which the water flows from the tap is the same when filling each container. Draw a possible water level vs. time graph for each container.





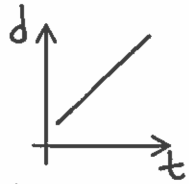




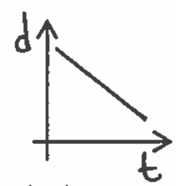
In which container will the water level rise fastest? graduated cylinder because it has the smallest diameter

Note: In a graph of displacement (ie. distance, height, or depth) vs time:

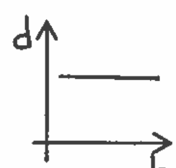
$$Speed = \left|\frac{\text{change in displacement}}{\text{change in time}}\right| = \left|\frac{\Delta d}{\Delta t}\right|$$



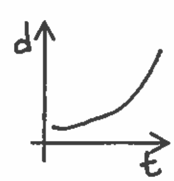
d increases as t increases



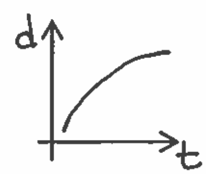
d decreases as t increases



d stays constant over time



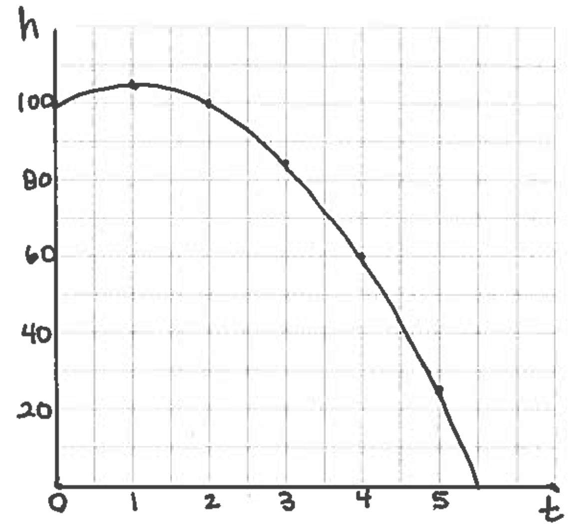
speed is increasing



speed is decreasing

Eg. 3 A student tosses a ball out of a window and its height, in metres, over time, in seconds, is recorded in the following table:

t	h	Δh	$\Delta^2 h$
0	100		
1	105	5	
2	100	−5	−10
3	85	−15	−10
4	60	−25	−10
5	25	−35	−10



a) Calculate the finite differences and determine what type of function will model this behavior. Graph the function.

Since the second differences are the same, the function is quadratic.

b) Determine the equation for this function.

Let the function be represented by $h(t)=at^2+bt+c$

From the table, when $t=0$, $h=100$

$100=a(0)^2+b(0)+c$

$\therefore c=100$

From table, $t=1$ $h=105$

$105=a(1)^2+b(1)+100$

$5=a+b$ ①

From table, $t=2$ $h=100$

$100=a(2)^2+b(2)+100$

$(\div 2)$ $0=4a+2b$

$0=2a+b$ ②

Subtract: $2a+b=0$ ②

$a+b=5$ ①

$a=-5$

Sub $a=-5$ in ①

$-5+b=5$

$b=10$

$\therefore h(t)=-5t^2+10t+100$

c) Using the table of values and the graph, describe what is happening with the speed of the ball over time.

- slows down
- pauses (max height)
- increases quickly as ball falls to ground

d) Determine the rate of change of displacement (height)

i) in the first second

$$\frac{\Delta d}{\Delta t}=\frac{d(1)-d(0)}{1-0}$$
$$=\frac{105-100}{1}$$
$$=5 \text{ m/s}$$

ii) between 1 and 3 seconds

$$\frac{\Delta d}{\Delta t}=\frac{d(3)-d(1)}{3-1}$$
$$=\frac{85-105}{2}$$
$$=\frac{-20}{2}=-10 \text{ m/s}$$

(negative indicates ball is falling)