Rational Exponents
↳ fractions

Property 1

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

$4^{\frac{1}{2}} \times 4^{\frac{1}{2}} = 4$ but $\sqrt{4} \times \sqrt{4}$
$= 2 \times 2$
$= 4$
$\therefore 4^{\frac{1}{2}} = \sqrt{4} = \sqrt[2]{4}$

Property 2

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$$

$a^{\frac{m}{n}} = a^m \times a^{\frac{1}{n}}$
$= (a^m)^{\frac{1}{n}}$
$= \sqrt[n]{a^m}$

$a^{\frac{m}{n}} = a^{\frac{1}{n}} \times a^m$
$= (a^{\frac{1}{n}})^m$
$= (\sqrt[n]{a})^m$

Ex. Simplify

a) $36^{\frac{1}{2}}$
$= \sqrt{36}$
$= 6$

b) $(-27)^{\frac{1}{3}}$
$= \sqrt[3]{-27}$
$= -3$

c) $(-8)^{-\frac{1}{3}}$
$= \left(\frac{1}{-8}\right)^{\frac{1}{3}}$
$= \frac{1}{\sqrt[3]{-8}}$
$= \frac{-1}{2}$

d) $27^{\frac{2}{3}}$
$= (\sqrt[3]{27})^2$
$= 3^2$
$= 9$

e) $4^{-\frac{5}{2}}$
$= \left(\frac{1}{4}\right)^{5/2}$
$= \frac{1}{(\sqrt{4})^5}$
$= \frac{1}{2^5}$
$= \frac{1}{32}$

f) $9^{-2.5}$
$= 9^{-5/2}$
$= \left(\frac{1}{9}\right)^{5/2}$
$= \frac{1}{(\sqrt{9})^5}$
$= \frac{1}{3^5}$
$= \frac{1}{243}$

g) $\left(\frac{25}{4}\right)^{-\frac{3}{2}}$
$= \left(\frac{4}{25}\right)^{3/2}$
$= \left(\sqrt{\frac{4}{25}}\right)^3$
$= \left(\frac{2}{5}\right)^3$
$= \frac{8}{125}$

h) $\sqrt[3]{\left(\sqrt{\sqrt{x^4}}\right)^2}$
$= \left(\left(\left(\left(x^4\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)^2\right)^{\frac{1}{3}}$
$= x^{\frac{2}{3}}$