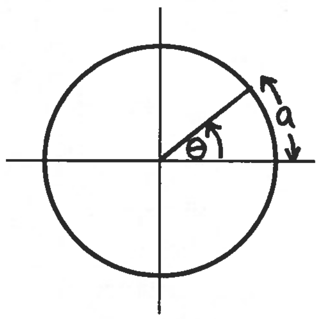
Radians

Angles are usually measured in *degrees*, but, mathematicians, and physicists saw a need for another measure that would simplify some calculations.

RADIANS are another method for measuring an angle. Radians are a pure number (ie. no units) so unlike degrees, they are more suited to certain mathematical calculations.

Recall: Given a circle of radius, *r*, and centre (0, 0):



Circumference of a circle: $C = 2\pi r$

Arc Length: $a = \theta r$

For <u>one complete rotation</u>, note that: arc length = circumference

$\therefore \theta r = 2\pi r$

$\theta = 2\pi \Rightarrow 2\pi$ radians in one complete rotation

Note: 1 radian = measure of an angle subtended at the centre of the circle by an arc length equal to the radius

In general,

$360° = 2\pi$ rad

$180° = \pi$ rad

$\boxed{1° = \frac{\pi}{180} \text{ rad}}$ $\boxed{1 \text{ rad} = \frac{180}{\pi} \text{ degrees}}$

Eg. 1 Convert to radians:

mult. by $\frac{\pi}{180}$

a) $135° = 135 \times \frac{\pi}{180}$
$= \frac{3\pi}{4}$

b) $30° = 30 \times \frac{\pi}{180}$
$= \frac{\pi}{6}$

c) $90° = 90 \times \frac{\pi}{180}$
$= \frac{\pi}{2}$

d) $45° = 45 \times \frac{\pi}{180}$
$= \frac{\pi}{4}$

e) $270° = 270 \times \frac{\pi}{180}$
$= \frac{3\pi}{2}$

f) $60° = 60 \times \frac{\pi}{180}$
$= \frac{\pi}{3}$

Eg. 2 Convert to degrees:

mult. by $\frac{180}{\pi}$

a) $\frac{3\pi}{2}$ rad
$= \frac{3\cancel{\pi}}{\cancel{2}} \times \frac{\cancel{180}^{90}}{\cancel{\pi}}$
$= 270°$

b) $\frac{5\pi}{12}$ rad
$= \frac{5\cancel{\pi}}{\cancel{12}} \times \frac{\cancel{180}^{15}}{\cancel{\pi}}$
$= 75°$

c) 4 rad
$= 4 \times \frac{180}{\pi}$] $4 \times 180 \div \pi$ ↑ use π button
$\doteq 229°$

Ex. 3 Evaluate exactly:

a) $\cos\left(\frac{\pi}{3}\right)$
$= \cos\left(\frac{\pi}{3} \times \frac{180}{\pi}\right)$
$= \cos 60°$
$= \frac{1}{2}$

b) $\sin\left(\frac{3\pi}{4}\right)$
$= \sin\left(\frac{3\pi}{4} \times \frac{180}{\pi}\right)$
$= \sin 135°$
$= \frac{1}{\sqrt{2}}$

c) $\tan\left(\frac{5\pi}{6}\right)$
$= \tan\left(\frac{5\pi}{6} \times \frac{180}{\pi}\right)$
$= \tan 150°$
$= -\frac{1}{\sqrt{3}}$

Graphing trig functions in radians:

a= amplitude
p= phase shift
q= vertical shift

Recall: $$y = a\cos\left[k\left(x-p\right)\right]+q$$

In degrees: $\boxed{\text{period} = \frac{360^\circ}{k}}$ In radians: $\boxed{\text{period} = \frac{2\pi}{k}}$

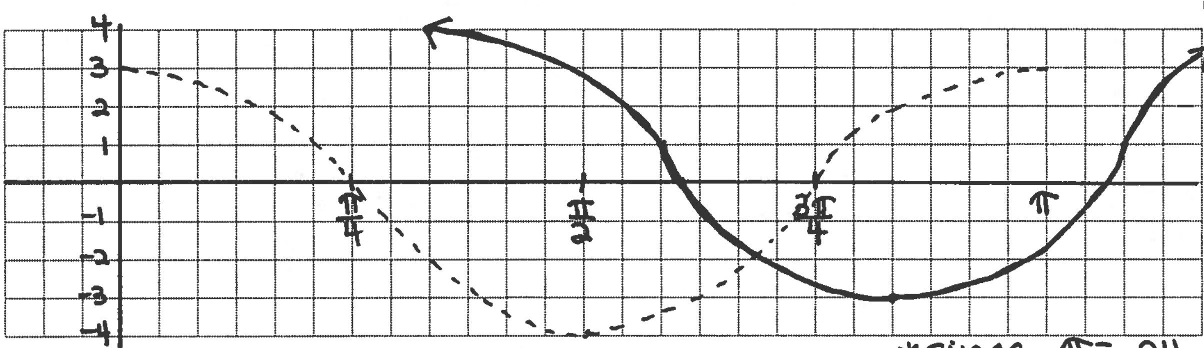
* [Note that the values for "a" and "q" are unaffected when the equation is given in radians.]

Eg. 4 Graph: $y = 3\cos\left[2\left(x-\frac{\pi}{3}\right)\right]+1$

$a=3$ period $= \frac{2\pi}{2} = \pi$

phase shift: right $\frac{\pi}{3}$
vert. shift: up 1

* Scale: Use a multiple of 12 squares to represent π.



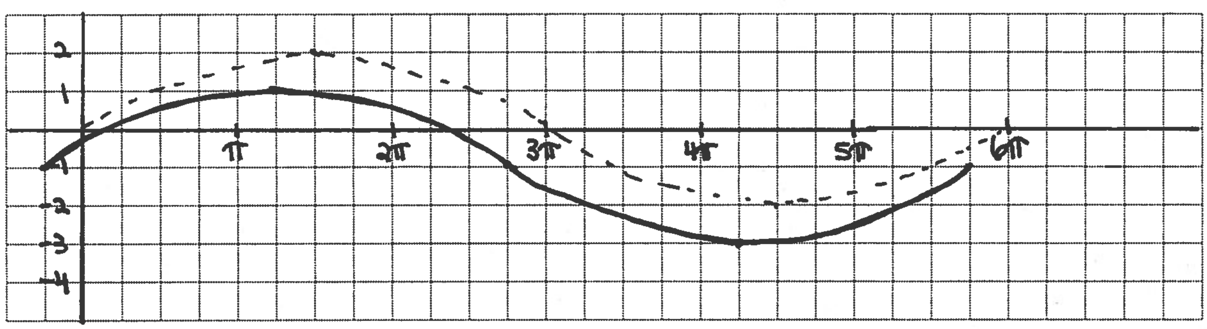
*since $\pi = 24$ squares
$\frac{\pi}{3} = \frac{24}{3} = 8$ squares

Eg. 5 Graph: $y = 2\sin\left[\frac{1}{3}\left(x+\frac{\pi}{4}\right)\right]-1$

$a=2$ period $= \frac{2\pi}{1/3} = 6\pi$

phase shift: left $\pi/4$
vert. shift: down 1

* scale: use $6\pi = 24$ squar



*since $\pi = 4$ squares
$\frac{\pi}{4} = 1$ square

Homework: pg. 334 # 1 – 4 (odds)
pg. 348 # 4
pg. 387 #7