The Quadratic Formula

Last day we looked at solving quadratic equations by completing the square.
Consider the general case:

Solve: $ax^2+bx+c=0$

Complete the square:

$$a\left(x^2+\frac{b}{a}x\right)+c=0$$

* Factor out "a"

$\frac{1}{2}$ of $\frac{b}{a} = \frac{b}{2a}$

$\left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$

$$a\left(\underbrace{x^2+\frac{b}{a}x+\frac{b^2}{4a^2}}-\frac{b^2}{4a^2}\right)+c=0$$

$$a\left(x+\frac{b}{2a}\right)^2-\frac{\cancel{a}b^2}{4a^{\cancel{2}}}+c=0$$

$$a\left(x+\frac{b}{2a}\right)^2=\frac{b^2}{4a}-c$$

* Bring extra terms to Right side

$$a\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a}$$

* get common denominator

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a}\times\frac{1}{a}$$

* divide both sides by "a"

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$$

* take $\sqrt{\ }$ of both sides

$$x+\frac{b}{2a}=\pm\frac{\sqrt{b^2-4ac}}{2a}$$

(Note: $\sqrt{4a^2}=2a$)

$$x=\frac{-b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$\boxed{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$$

Quadratic Formula

use to solve any quadratic equation of the form $ax^2+bx+c=0$

Eg 1) Solve using the quadratic formula:

$$2x^2+5x-12=0$$

$a=2$ $\quad$ $b=5$ $\quad$ $c=-12$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

sub. $a=2, b=5, c=-12$

$$x=\frac{-(5)\pm\sqrt{(5)^2-4(2)(-12)}}{2(2)}$$

$$x=\frac{-5\pm\sqrt{25+96}}{4}$$

$$x=\frac{-5\pm\sqrt{121}}{4}$$

$$x=\frac{-5\pm 11}{4}$$

$$x=\frac{-5+11}{4} \quad \text{or} \quad x=\frac{-5-11}{4}$$

$$x=\frac{6}{4}=\frac{3}{2} \qquad x=\frac{-16}{4}=-4$$

Eg 2) Solve: $\frac{x^2+4x}{3}=\frac{-1}{4}$

First multiply by 12 to eliminate fractions:

$$12\left(\frac{x^2+4x}{3}\right)=12\left(\frac{-1}{4}\right)$$

$$4(x^2+4x)=3(-1)$$

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$$4x^2+16x+3=0$$

Sub $a=4$ $b=16$ $c=3$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-16\pm\sqrt{(16)^2-4(4)(3)}}{2(4)}$$

$$=\frac{-16\pm\sqrt{208}}{8}$$

simplify radical

$$=\frac{-16\pm 4\sqrt{13}}{8}$$

$\div$ each term by 4

$$x=\frac{-4\pm\sqrt{13}}{2}$$

Eg 3) Solve, if possible: $3x^2+10=0$

Sub $a=3$ $b=\underline{\underline{0}}$ $c=10$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(0)\pm\sqrt{(0)^2-4(3)(10)}}{2(3)}$$

$$x=\frac{0\pm\sqrt{-120}}{6}$$

can't take $\sqrt{}$ of negative number

$\therefore$ there are no real solutions (parabola doesn't cross the x-axis)

The Discriminant

In the quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

the part under the square root sign (b^2-4ac) is called the Discriminant.

Note:

1) If $\boxed{b^2-4ac<0}$, then the quadratic equation has no real solutions (parabola doesn't cross the x-axis)

2) If $\boxed{b^2-4ac>0}$, then the quadratic equation has two real distinct solutions (parabola has two different x-int.)

3) If $\boxed{b^2-4ac=0}$, then the quadratic equation has two real equal solutions (parabola has its vertex on the x axis)

Eg. Determine the nature of the roots given

$$2x^2-5x+8=0$$

$$\begin{aligned} b^2-4ac &= (-5)^2-4(2)(8) \\ &= 25-64 \\ &= -39 < 0 \end{aligned}$$

Since $b^2-4ac<0$ then the parabola has no real roots.

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