MI M 2DI

Date: _____

PROBLEM SOLVING WITH QUADRATIC EQUATIONS

Eg. 1 The height of one kick of a soccer ball related to the horizontal distance from the place on the ground where it was kicked can be modeled by the function $h = -0.025d^2 + d$, where h is the height, in metres, and d is the horizontal distance, in metres. Determine the horizontal distance travelled by the ball from the time it is kicked until it hits the ground.

* Solve $h = -0.025d^2 + d$ when $h = 0$ (h=0 at ground level)

$0 = -0.025d^2 + d$

$0 = d(-0.025d + 1)$

$d = 0$ $\quad -0.025d + 1 = 0$

$1 = 0.025d$

$\frac{1}{.025} = d$

$d = 40$

distance travelled is 0 when the ball is first kicked

distance travelled when the ball returns to ground level

∴ The ball travels a horizontal distance of 40 m.

Eg. 2 The sum of the squares of two consecutive integers is 421. Find the numbers.

Let x be one number. Let y be the other number.

Since the numbers are consecutive, $y = x + 1$

Given: $x^2 + y^2 = 421$ (sum of the squares)

Sub $y = x + 1$:

$x^2 + (x+1)^2 = 421$

Simplify: $x^2 + x^2 + 2x + 1 = 421$

$2x^2 + 2x + 1 - 421 = 0$

$2x^2 + 2x - 420 = 0$

Solve: $2(x^2 + x - 210) = 0$

$2(x + 15)(x - 14) = 0$

$x + 15 = 0$ $\quad x - 14 = 0$

$x = -15$ $\quad x = 14$

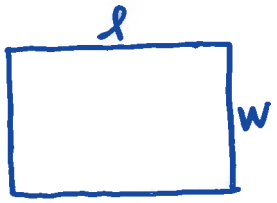
When $x = -15$, $y = -15 + 1 = -14$

When $x = 14$, $y = 14 + 1 = 15$

∴ The numbers are 14 and 15 OR -14 and -15.

Eg. 3 The width of a rectangle is 5 m less than the length. The area is 84 m^2. Determine the width and the length.

Let w be the width and l be the length.



Given $w = l - 5$

Since $A = l \cdot w = 84$

$l(l-5) = 84$

$l^2 - 5l - 84 = 0$

$(l-12)(l+7) = 0$

$l - 12 = 0$ or $l + 7 = 0$

$l = 12$ or $l = -7$ inadmissible

length can't be negative so don't use this value

When $l = 12$

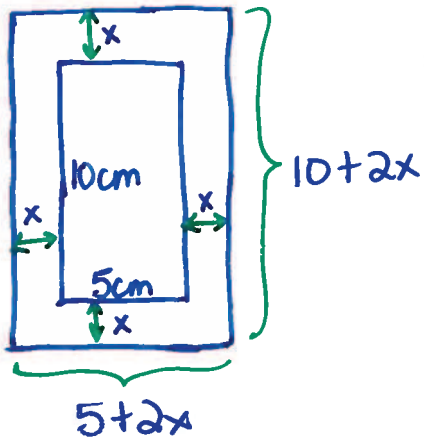
$w = 12 - 5$

$w = 7$

∴ The width should be 7 m and the length should be 12 m.

Eg. 4 A picture that measures 5 cm by 10 cm is to be surrounded by a mat before being framed. The width of the mat must be the same on all sides of the picture. The area of the mat is to be twice the area of the picture. What is the width of the mat?

Let x be the width of the mat.



Area of Picture = $5 \times 10 = 50\ cm^2$

Area of Photo Mat = $100\ cm^2$ (twice area of photo)

∴ Area of Picture and Mat together = $150\ cm^2$

Equation:

$(5+2x)(10+2x) = 150$

$50 + 10x + 20x + 4x^2 = 150$

$4x^2 + 30x + 50 - 150 = 0$

$4x^2 + 30x - 100 = 0$

Solve: $2(2x^2 + 15x - 50) = 0$

$2(2x-5)(x+10) = 0$

$2x - 5 = 0$ or $x + 10 = 0$

$2x = 5$ or $x = -10$

$x = \frac{5}{2}$ mat width can't be negative

∴ The width of the photo mat is $\frac{5}{2}$ cm (or 2.5 cm).