MCV 4U1 Feb 23/12

The Product Rule

When a function can be expressed as the product of two other functions, ie. $f(x) = g(x) \cdot k(x)$, we can determine the derivative using the Product Rule: $\boxed{f'(x) = g'(x) \cdot k(x) + g(x) \cdot k'(x)}$

(proof later)

Eg. Differentiate and simplify.

(a) $y = \underbrace{(3x^3 - 8x^2)}_{g(x)} \underbrace{(x^2+5)}_{k(x)}$

$y' = \underbrace{(9x^2 - 16x)}_{g'(x)} \underbrace{(x^2+5)}_{k(x)} + \underbrace{(3x^3 - 8x^2)}_{g(x)} \underbrace{(2x)}_{k'(x)}$

Simplify.

$y' = 9x^4 + 45x^2 - 16x^3 - 80x + 6x^4 - 16x^3$

$= 15x^4 - 32x^3 + 45x^2 - 80x$

(b) $f(x) = \sqrt{x}\,(2-x)$

$f'(x) = \left(\frac{1}{2}x^{-1/2}\right)(2-x) + \left(x^{1/2}\right)(-1)$

$= x^{-1/2} - \frac{1}{2}x^{1/2} - x^{1/2}$

$= x^{-1/2} - \frac{3}{2}x^{1/2}$

$f'(x) = \frac{1}{\sqrt{x}} - \frac{3}{2}\sqrt{x}$

Eg2 Find the equation of the tangent to the curve $y = x(\sqrt{x}+4)$ at $x=4$.

$$\frac{dy}{dx} = (1)(\sqrt{x}+4) + x\left(\frac{1}{2}x^{-1/2}\right)$$
$$= \sqrt{x}+4 + \frac{1}{2}x^{1/2}$$
$$= \sqrt{x}+4 + \frac{1}{2}\sqrt{x}$$
$$= \frac{3}{2}\sqrt{x} + 4$$

this notation means sub $x=4$ in to $\frac{dy}{dx}$

$$m_{tan} = \left.\frac{dy}{dx}\right|_{x=4} = \frac{3}{2}\sqrt{4} + 4$$
$$= \frac{3}{2}(2) + 4$$
$$= 7$$

At $x=4$, $y = 4(\sqrt{4}+4)$
$$= 4(2+4)$$
$$= 24$$

$\therefore$ Equation of tangent at $(4,24)$ with $m=7$

$$\frac{7}{1} = \frac{y-24}{x-4}$$

$$7x - 28 = y - 24$$

$$7x - 4 = y$$

HW: pg 145 #1 cde, 4 aceg, 5, 6b