MCB 4UI – *One-Sided Limits*

Consider the piece-wise function: $f(x)=\begin{cases} x+1 & \text{if } x>0 \\ x-1 & \text{if } x\le 0 \end{cases}$

What is the $\lim_{x\to 0} f(x)$? Lets investigate!

Approaching from left

x	$f(x)$
–0.5	–1.5
–0.1	–1.1
–0.01	–1.01
–0.001	–1.001

0   –1

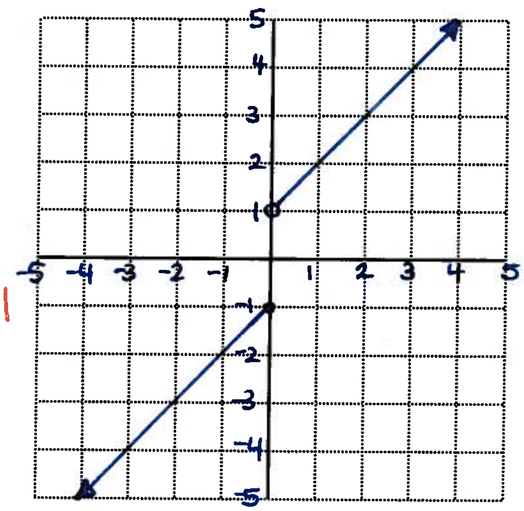
$\lim_{x\to 0^-} f(x) = -1$

Approaching from right

x	$f(x)$
0.5	1.5
0.1	1.1
0.01	1.01
0.001	1.001

0   1

$\lim_{x\to 0^+} f(x) = 1$



$\because \lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$

$\therefore \lim_{x\to 0} f(x)$ does not exist

(i.e. the limit of the function from the left and from the right must be equal for the limit to exist at that particular x-value!)

E.g. *sfpp* Evaluate the following limits:

a) $\lim_{x\to 0}\sqrt{x}$

$\lim_{x\to 0^+}\sqrt{x} = 0$

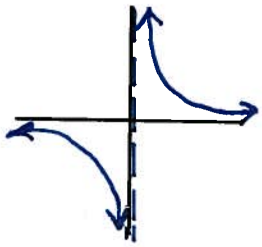
$\lim_{x\to 0^-}\sqrt{x} = \text{undefined}$

* can not approach $x=0$ from left

$\therefore \lim_{x\to 0}\sqrt{x}$ does not exist (DNE)

b) $\lim_{x\to 0}\frac{1}{x}$

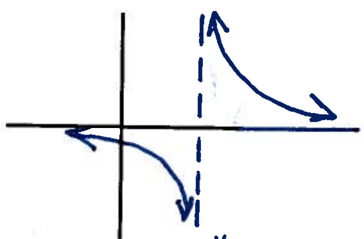
$\lim_{x\to 0^+}\frac{1}{x} = \infty$ $\lim_{x\to 0^-}\frac{1}{x} = -\infty$



$\therefore \lim_{x\to 0}\frac{1}{x} = \text{DNE}$

c) $\lim_{x\to 4}\frac{1}{x-4}$

$\lim_{x\to 4^+}\frac{1}{x-4} = +\infty$ $\lim_{x\to 4^-}\frac{1}{x-4} = -\infty$



$\therefore \lim_{x\to 4}\frac{1}{x-4} = \text{DNE}$

Continuity

A function $f(x)$ is continuous at a pt. $x=a$ if:

(1) $f(a)$ is defined [ie. there is a y-value at $x=a$]

(2) $\lim_{x\to a} f(x)$ exists [ie. $\lim_{x\to a^-} f(x) = L = \lim_{x\to a^+} f(x)$]

(3) $f(a) = \lim_{x\to a} f(x)$ [the y-value equals the value of the limit]