MPM 2D Date:

Radicals Review

Recall: $\sqrt{25} = 5$ because $5^2 = 25$
Since the square root of 25 is a whole number, we refer to 25 as a ***perfect square***.
Most numbers are not perfect squares since their square roots are a decimal.
ie. $\sqrt{3} \doteq 1.7$ (rounded) so 3 is not a perfect square

Usually, rather than evaluating such roots as a decimal, we write them as a radical in simplified form.

Entire radicals are written entirely under the radical sign $(\sqrt{\ })$ eg. $\sqrt{11}$, $\sqrt{90}$, etc.

Mixed radicals have a whole number part and a radical part eg. $\underbrace{2\sqrt{3}}_{\text{means } 2\times\sqrt{3}}$, $5\sqrt{10}$, etc.

Steps to simplify a radical:

1. Find the largest perfect square that divides evenly into the number under the radical sign.
2. Write the number under the radical sign as the product of the perfect square times another number.
3. Take the square root of the perfect square and write this as a whole number in front of the radical sign while leaving the other number under the radical sign.

Note: In simplified form, no perfect square (other than 1) divides into the number under the radical sign.

Eg. 1 Simplify as a mixed radical.

a) $\sqrt{45}$
$= \sqrt{9\times5}$
$= \sqrt{9}\times\sqrt{5}$
$= 3\sqrt{5}$

b) $\sqrt{98}$
$= \sqrt{49\times2}$
$= \sqrt{49}\times\sqrt{2}$
$= 7\sqrt{2}$

c) $5\sqrt{48}$
$= 5\sqrt{16\times3}$
$= 5(\sqrt{16}\times\sqrt{3})$
$= 5(4\sqrt{3})$
$= 20\sqrt{3}$ (multiply whole number parts together)

Eg. 2 Write as an entire radical.

a) $5\sqrt{3}$
$= \sqrt{25}\times\sqrt{3}$
$= \sqrt{75}$

b) $2\sqrt{7}$
$= \sqrt{4}\times\sqrt{7}$
$= \sqrt{28}$

Operations with Radicals:

1) *Adding & Subtracting Radicals*

- Start by simplifying each radical term in the expression
- Combine "like" radicals by adding/subtracting the whole number coefficients

Eg. 3 Simplify. coefficient is 1

a) $2\sqrt{3}+4\sqrt{3}-\sqrt{3}$

$= 5\sqrt{3}$

b) $\sqrt{50}+2\sqrt{32}$

$= \sqrt{25\times 2}+2\sqrt{16\times 2}$

$= 5\sqrt{2}+2(4\sqrt{2})$

$= 5\sqrt{2}+8\sqrt{2}$

$= 13\sqrt{2}$

c) $3\sqrt{4x}+7\sqrt{25x}-5\sqrt{12x}$

$= 3(2\sqrt{x})+7(5\sqrt{x})-5\sqrt{4\cdot 3x}$

$= 6\sqrt{x}+35\sqrt{x}-5(2\sqrt{3x})$

$= 6\sqrt{x}+35\sqrt{x}-10\sqrt{3x}$

$= 41\sqrt{x}-10\sqrt{3x}$ (note: these are not like terms)

2) *Multiplying Radicals*

- Multiply whole number parts together and radical parts together, then simplify overall expression.

Eg. 4 Simplfy.

a) $4\sqrt{3}\times 2\sqrt{2}$

$= 8\sqrt{6}$

(4×2) $(\sqrt{3}\times\sqrt{2})$

b) $3\sqrt{2}(\sqrt{6}+2\sqrt{7})$ *use distributive property

$= 3\sqrt{12}+6\sqrt{14}$

$= 3\sqrt{4\times 3}+6\sqrt{14}$

$= 3(2\sqrt{3})+6\sqrt{14}$

$= 6\sqrt{3}+6\sqrt{14}$

3) *Dividing Radicals*

- Divide whole number parts, and divide radical parts, then simplify overall expression.
- If there is a radical left in the denominator that won't divide into the numerator, we rationalize the denominator by multiplying top & bottom of the expression by the radical in the denominator.

Eg. 5 Divide. Rationalize the denominator, if necessary.

a) $\dfrac{18\sqrt{12}}{3\sqrt{6}}$

$= 6\sqrt{2}$

$18\div 3$ $\sqrt{12}\div\sqrt{6}$

b) $\dfrac{8}{\sqrt{2}}$ *Rationalize

$= \dfrac{8}{\sqrt{2}}\cdot\dfrac{\sqrt{2}}{\sqrt{2}}$

$= \dfrac{8\sqrt{2}}{2}$

$= 4\sqrt{2}$

c) $\dfrac{\sqrt{12}-3\sqrt{15}}{\sqrt{3}}$ *divide radical on bottom into each term on top

$= \sqrt{4}-3\sqrt{5}$

$= 2-3\sqrt{5}$