Equation of a Parabola

The general vertex form of the equation of a parabola is:

$$y = \pm a(x-h)^2 + k$$

- $\pm$: direction
- a: vertical stretch or compression
- h: shift left/right
- k: shift up/down

The vertex is given by (h, k)

Eg① Write the equation of a parabola given:

(a) vertex $(-4, 0)$; $a = -3$ (h, k)

*Sub $h=-4$, $k=0$, $a=-3$ into above general equation.

$\therefore y = -3(x+4)^2$

$x-(-4)$; +0 on end

(b) vertex $(3, 2)$; $a = \frac{1}{2}$

$\therefore y = \frac{1}{2}(x-3)^2 + 2$

(c) vertex $(-3, 6)$ through the point $(-2, 10)$ (h, k)

Sub $h=-3$, $k=6$ into general equation.

$y = a(x+3)^2 + 6$

Sub $x=-2$, $y=10$ →

$10 = a(-2+3)^2 + 6$

$10 = a(1)^2 + 6$

$10 = a + 6$

$a = 4$

*Since we don't know "a" substitute point $(-2, 10)$ for (x, y) then solve for "a".

$\therefore y = 4(x+3)^2 + 6$

Eg② Determine the value of k so that the graph of $y=-3(x+2)^2+k$ passes through the point $(-3,-7)$.

Sub. pt $(-3,-7)$ for (x,y) ($x=-3$, $y=-7$)

$y=-3(x+2)^2+k$
$-7=-3(-3+2)^2+k$
$-7=-3(-1)^2+k$
$-7=-3(1)+k$
$-7=-3+k$
$-7+3=k$
$-4=k$

$\therefore y=-3(x+2)^2-4$

Eg③ Write the equation of a parabola that is congruent to $y=2x^2$ with a minimum pt at $(-2,4)$.

vertex $h=-2, k=4$

same size $\Rightarrow$ same value for "a" $(\pm)$
$\therefore a=2$

$\therefore y=2(x+2)^2+4$