Unit 7 - Functions and Rates of Change

Function - a relation where every x-value has a unique y-value

Domain: $\{x \in \mathbb{R}\}$ Set of all x value

Range: $\{y \in \mathbb{R}\}$ Set of all y values

Function notation

$y = x^2 \Rightarrow f(x) = x^2$

A function can be represented as:

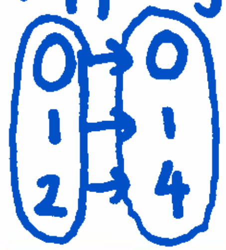
1) Equations
eg. $y = x^2$

2) Graphs

3) Table of Values

x	y
0	0
1	1
2	4

4) Mapping diagrams



Piecewise Functions

a function which has different algebraic expressions for different parts of the domain

eg: $f(x) = \begin{cases} 1, & x < -2 \\ -x^2+5, & -2 \leq x < 3 \\ -2x+8, & x \geq 3 \end{cases}$

$>, < = \circ$ open

$\geq, \leq = \bullet$ closed

Combining Functions

General Operations

$(f+g)(x) = f(x) + g(x)$

$(f-g)(x) = f(x) - g(x)$

$(fg)(x) = f(x) \cdot g(x)$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \; g(x) \neq 0$

Domain is where the two functions overlap

ex.

$\{x \in \mathbb{R} \mid -1 \leq x \leq 1\}$

Symmetry:

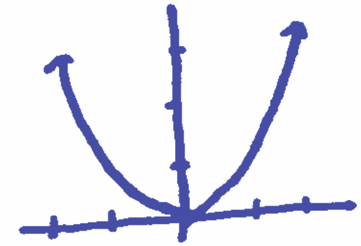
Even Function - symmetric about the y-axis

$F(-x) = f(x)$

i.e: $f(x) = x^2$

$f(-x) = (-x)^2$

$f(-x) = x^2$



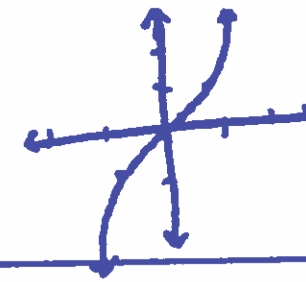
Odd Function - symmetric about the origin

$F(-x) = -f(x)$

i.e: $f(x) = x^3$

$f(-x) = (-x^3)$

$f(-x) = -x^3$



Inverse: - reflection of f(x) through line $y = x$

1. Switch the x and y value
2. Solve for y

i.e: $f(x) = 4x - 3$

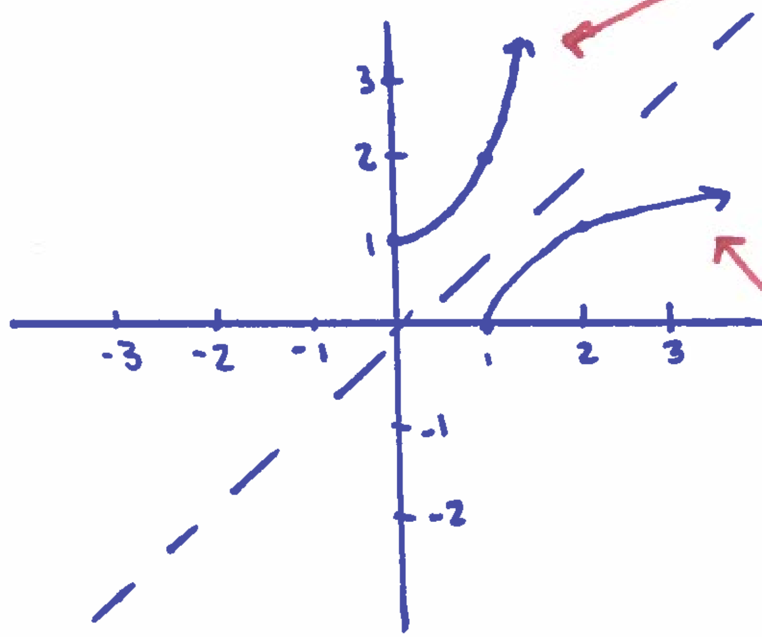
$y = 4x - 3$

$x = 4y - 3$

$x + 3 = 4y$

$\frac{x+3}{4} = y$

$\therefore f^{-1}(x) = \frac{x+3}{4}$



$\boxed{y = x^2 + 1}$, $x \geq 0$

$x = y^2 + 1$

$x - 1 = y^2$

$\boxed{\sqrt{x-1} = y}$ $y \geq 0$

Absolute Value - refers to distance from zero, $|x|$

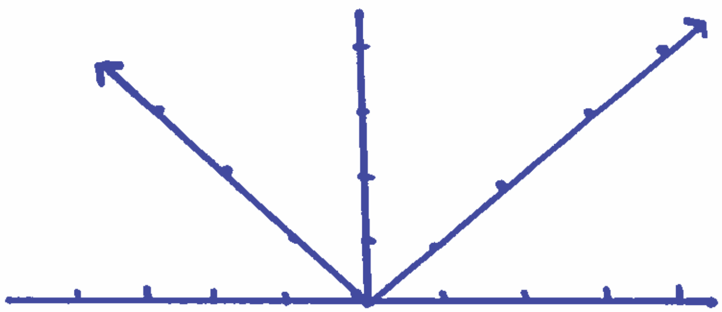
i.e: $|-4-(-5)|$
$= |-4+5|$
$= |1|$
$= 1$

Solve 2 cases:
i.e: $|2x-1|=8$
$2x-1=8$ or $2x-1=-8$
$x = 9/2$ $\quad$ $x = -7/2$

State the sol'n set for:
$|2x+1|>5$

$2x+1>5$ or $-(2x+1)>5$
$2x>4$ $\quad$ $2x+1<-5$
$x>2$ $\quad$ $2x<-6$
$x<-3$

Graph $y=|x|$:



Composition of Functions - given 2 functions f(x) and g(x), solve by substitution one into the other

$\circ \longrightarrow$ composition

$(f \circ g)(x) = f(g(x))$

i.e: $f(x)=\sqrt{x}$ $\quad$ $g(x)=x^2-\frac{1}{x}$
$D=\{x \in \mathbb{R} \mid x \geq 0\}$ $\quad$ $D=\{x \in \mathbb{R} \mid x \neq 0\}$

$f \circ g = f(g(x))$
$= \sqrt{x^2-\frac{1}{x}}$
$x^2-\frac{1}{x} > 0$
$x^3>1$
$x>1$

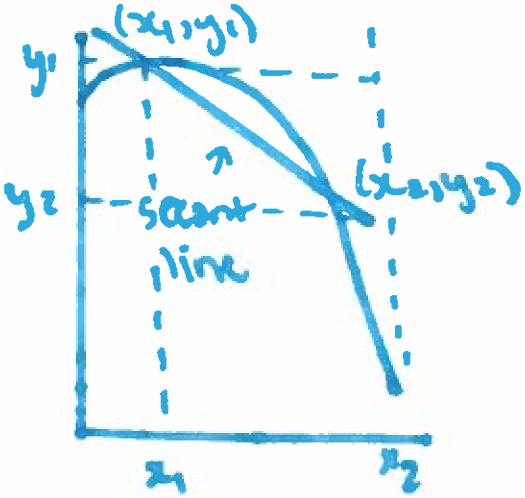
$g \circ f = g(f(x))$
$= (\sqrt{x})^2 - \frac{1}{x}$
$= x - \frac{1}{x}$
$x \neq 0$
$x>0$

Note: the domain of f(x) is the range of g(x).

Average Rate of Change: * TWO VALUES GIVEN!

Change in quantity of dependent value divided by independent variable.

AROC = $\dfrac{\text{change in } Y}{\text{change in } x}$



$\therefore \dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1}$

Instantaneous Rate of Change: * ONE VALUE IS GIVEN!

Find speed at exact point:

Example: Determine IROC at $t = 2$ hours

$$\text{IROC} = \lim_{h \to 0} \frac{c(2+h) - c(2)}{2+h-2}$$

$$= \frac{\left(\frac{20+10h}{h^2+4h+9}\right) - \left(\frac{20}{9}\right)}{h}$$

$$= \frac{9(20+10h) - 20(9+4h+h^2)}{9(9+4h+h^2)} \div h$$

$c(2) = \dfrac{10t}{t^2+5}$

$= \dfrac{20}{9}$

$c(2+h) = \dfrac{10(2+h)}{(2+h)^2+5}$

$$\frac{180 + 90h - 180 - 80 - 20h^2}{81 + 36h + 9h^2}$$

$\lim h \to 0$

$= \dfrac{10(0) - 20(0)^2}{81 + 36h + 9h^2}$

$= \dfrac{10}{81}$

Practice Questions:

Find Domain & Range: #1

a) $f(x) = 2x - 13$ b) $f(x) = \frac{x}{3x-5}$

Graph:

$$f(x) = \begin{cases} x & \text{if } x < -1 \\ x^3 & \text{if } -1 < x \leq 1 \\ (x-2)^2 & \text{if } x > 1 \end{cases}$$

Find Inverse: #2

a) $f(x) = x^3 + 5$ b) $f(x) = \frac{2x-1}{3x+4}$

State if odd/even/neither #3

a) $f(x) = 2x^4 + 3x^2$ b) $f(x) = \frac{1}{x^3+1}$

Solve for: #4

a) $|5x+4| > 3$ b) $|4x-3| \leq 2$

For $f(x) = \frac{x}{1-x^2}$ find: **#5**

a) $f+g$ b) fg c) $\frac{f}{g}$ d) $f \circ g$ e) $g \circ f$

#6 If $h = f \circ g$ and $h(x) = (2x-1)^3$ find functions for f and g

(Flare shot from deck) **#7**

Time	Height
0	12
4	332
8	492
12	492
16	332

a) What type of function
b) Determine function
c) If fired 2m from deck to boat then how high up was the deck

#8 Height in meters of a ball above ground can be modelled: $h(t) = -5t^2 + 20t$

a) Find the average speed between 1 and 3 sec
b) Find instant speed of ball at 3 sec (use given values or limits)

Unit 7 Practice Questions - Solutions : Odd Numbers

#1 Find Domain and Range:

a) $f(x) = 2x - 13$

$D = \{x \in \mathbb{R}\}$

$R = \{y \in \mathbb{R}\}$

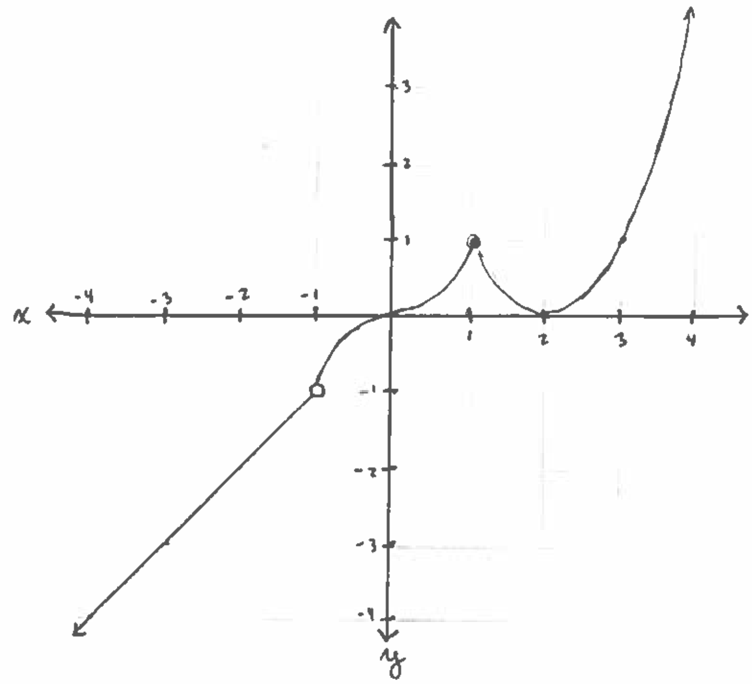
b) $f(x) = \frac{x}{3x-5}$ ★ Denominator cannot equal zero

$D = \{x \in \mathbb{R} \mid x \neq \frac{5}{3}\}$

$R = \{y \in \mathbb{R} \mid y \neq \frac{1}{3}\}$ ★ Ratio of coefficients of x $\frac{1x}{3x} = \frac{1}{3}$ when degrees of numerator and denominator are the same

Graph :

$$f(x) = \begin{cases} x & \text{if } x < -1 \\ x^3 & \text{if } -1 < x \leq 1 \\ (x-2)^2 & \text{if } x > 1 \end{cases}$$



#3 State if Even/Odd/Neither

a) $f(x) = 2x^4 + 3x^2$

sub $-x$ for x

$= 2(-x)^4 + 3(-x)^2$

$= 2x^4 + 3x^2$

EVEN because it is the same as the original function

b) $f(x) =$

$= \frac{1}{x^2+1}$

$= \frac{1}{(-x)^3+1}$

$= \frac{1}{-x^3+1}$

NEITHER because it is not the same as the original function, or multiplied by -1

#5 ps 47 7a

For $f(x) = x^2 - 4x + 3$ $g(x) = x - 1$

Find: a) $f+g$ b) fg c) $\frac{f}{g}$ d) $f \circ g$ e) $g \circ f$

a) $(x^2 - 4x + 3) + (x - 1)$

$= x^2 - 3x + 2$

b) $(x^2 - 4x + 3)(x - 1)$

$= x^3 - 4x^2 + 3x - x^2 + 4x - 3$

$= x^3 - 5x^2 + 7x - 3$

c) $\frac{(x^2 - 4x + 3)}{(x - 1)}$

$= \frac{\cancel{(x-1)}(x-3)}{\cancel{(x-1)}}$

$= x - 3$

d) $(x-1)^2 - 4(x-1) + 3$

$= x^2 - 2x + 1 - 4x + 4 + 3$

$= x^2 - 6x + 8$

e) $(x^2 - 4x + 3) - 1$

$= x^2 - 4x + 2$

Unit 1: Practice Questions Solutions

2. a) $f(x) = x^3 + 5$

$y = x^3 + 5$

$x = y^3 + 5$

$x - 5 = y^2$

$\sqrt[3]{x-5} = y$

b) $f(x) = \frac{2x-1}{3x+4}$

$y = \frac{2x-1}{3x+4}$

$x = \frac{2y-1}{3y+4}$

$3xy + 4x = 2y - 1$

$4x + 1 = -3xy + 2y$

$4x + 1 = y(-3x + 2)$

$\frac{4x+1}{2-3x} = y$

4. a) $|5x+4| > 3$

$5x + 4 > 3$ or $-5x - 4 > +3$

$5x > -1$ $\quad -5x > +7$

$x > -\frac{1}{5}$ $\quad x < -\frac{7}{5}$

$\therefore x > -\frac{1}{5} \cup x < -\frac{7}{5}$

b) $|4x-3| \leq 2$

$4x - 3 \leq 2$ or $-4x + 3 \leq 2$

$4x \leq 5$ $\quad -4x \leq -1$

$x \leq \frac{5}{4}$ $\quad x \geq \frac{1}{4}$

$\therefore x \geq \frac{1}{4} \cup x \geq \frac{5}{4}$

6. $h = f \circ g$ $\quad h = (2x-1)^3$

$h = f(g(x))$

$f = x^3$ $\quad g = 2x - 1$

8. $h(t) = -5t^2 + 20t$

a) $h(1) = -5(1)^2 + 20(1)$

$= -5 + 20$

$h(1) = 15$

$h(3) = -5(3)^2 + 20(3)$

$= -45 + 60$

$= 15$

$AROC = \frac{\Delta h}{\Delta t}$

$= \frac{15-15}{3-1}$

$\boxed{AROC = 0 m/s}$

b) $h(3) = 15$

$h(3+h) = -5(3+h)^2 + 20(3+h)$

$= -5(3+h)(3+h) + 20(3+h)$

$= -5(h^2 + 6h + 9) + 60 + 20h$

$= -5h^2 - 30h - 45 + 60 + 20h$

$= -5h^2 - 10h + 15$

$\lim_{h \to 0} \frac{h(3+h) - h(3)}{3 + h - 3}$

$= \frac{-5h^2 - 10h + 15 - 15}{h}$

$= \frac{-5h^2 - 10h}{h}$

$= \frac{-5h(h+2)}{h}$ $\quad$ * sub h=0

$= -5(h+2)$

$= -5(2)$

$\boxed{IROC = -10 m/s}$

7.

Time	Height	Δ_1	Δ_2
0	12	320	-160
4	332	160	-160
8	492	0	-160
12	492	-160	
16	332		

a) quadratic

b) $12 = a(0)^2 + b(0) + c$

$\boxed{12 = c}$

$332 = a(4)^2 + b(4) + c$

$332 = 16a + 4b + c$ ①

$332 - 12 = 16a + 4b$

$320 = 16a + 4b$ ②

$492 = a(8)^2 + b(8) + 12$

$480 = 64a + 8b$ ③

~~Multiply ② × 2:~~

~~$640 = 32a + 8b$~~

~~Subtract ② from ③:~~

~~$480 = 64a + 8b$~~

~~$-640 = 32a + 8b$~~

~~$-240 = 32a$~~

~~$\boxed{-7.5 = a}$~~

SORRY!

~~Sub $a = -7.5$ & $c = 12$ into ①~~

~~$332 = -7.5(16) + 4b + 12$~~

~~$320 = -120 + 4b$~~

~~$440 = 4b$~~

$110 = b$

$80 = 4a + b$ ②

$60 = 8a + b$ ③

Subtract ② from ③

$80 = 4a + b$

$- \; 60 = 8a + b$

$20 = -4a$

$\boxed{-5 = a}$

Sub $-5 = a$ and $12 = c$ into ②

$80 = 4(-5) + b$

$\boxed{100 = b}$

$\boxed{\therefore\ y = -5x^2 + 100b + 12}$

c) $h = -5t^2 + 100t + 12$

deck height $= 12$

deck height $= 12 - 2m$

$= 10m$