Unit 6: Trig II

Basic Identities:

$\tan x = \frac{\sin x}{\cos x}$ $\quad \cot x = \frac{\cos x}{\sin x}$ $\quad \csc x = \frac{1}{\sin x}$ $\quad \sec x = \frac{1}{\cos x}$

Pythagorean Identities:

$\sin^2 x + \cos^2 x = 1$

$1 + \cot^2 x = \csc^2 x$

$\tan^2 x + 1 = \sec^2 x$

Variations:

$\sin^2 x = 1 - \cos^2 x$ $\quad \cos^2 x = 1 - \sin^2 x$

$\cot^2 x = \csc^2 x - 1$ $\quad \csc^2 x - \cot^2 x = 1$

$\tan^2 x = \sec^2 x - 1$ $\quad \sec^2 x - \tan^2 x = 1$

Sum/Difference Formulas:

$\sin(x+y) = \sin x \cos y + \cos x \sin y$

$\sin(x-y) = \sin x \cos y - \cos x \sin y$

$\cos(x+y) = \cos x \cos y - \sin x \sin y$

$\cos(x-y) = \cos x \cos y + \sin x \sin y$

$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$

$\tan(x-y) = \frac{\tan x - \tan y}{[illegible]}$

Double Angle Formulas:

$\sin 2x = 2 \sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$\cos 2x = 2\cos^2 x - 1$

$\cos 2x = 1 - 2\sin^2 x$

$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$

Related/Corelated Identities:

$\frac{\pi}{2}$

π		$0/2\pi$
	$\sin(\pi - x) = \sin x$ $\cos(\pi - x) = -\cos x$ $\tan(\pi - x) = -\tan x$	$\sin(-x) = -\sin x$ $\cos(-x) = \cos x$ $\tan(-x) = -\tan x$
	$\sin(\pi + x) = -\sin x$ $\cos(\pi + x) = -\cos x$ $\tan(\pi + x) = \tan x$	$\sin(2\pi - x) = -\sin x$ $\cos(2\pi - x) = \cos x$ $\tan(2\pi - x) = -\tan x$

$\frac{3\pi}{2}$

$\frac{\pi}{2}$

π		
	$\sin\left(\frac{\pi}{2} + x\right) = \cos x$ $\cos\left(\frac{\pi}{2} + x\right) = -\sin x$ $\tan\left(\frac{\pi}{2} + x\right) = -\cot x$	$\sin\left(\frac{\pi}{2} - x\right) = \cos x$ $\cos\left(\frac{\pi}{2} - x\right) = \sin x$ $\tan\left(\frac{\pi}{2} - x\right) = \cot x$
	$\sin\left(\frac{3\pi}{2} - x\right) = -\cos x$ $\cos\left(\frac{3\pi}{2} - x\right) = -\sin x$ $\tan\left(\frac{3\pi}{2} - x\right) = \cot x$	$\sin\left(\frac{3\pi}{2} + x\right) = -\cos x$ $\cos\left(\frac{3\pi}{2} + x\right) = \sin x$ $\tan\left(\frac{3\pi}{2} + x\right) = -\cot x$

$\frac{3\pi}{2}$

Examples:

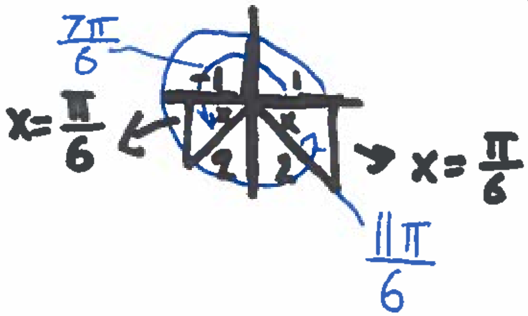
Solving Trig Equations:

Given $0 \le x \le 2\pi$

a) $2\sin x + 1 = 0$

$\sin x = \frac{-1}{2}$

※ $\sin x$ is only negative in Quad 3 & 4.



Sum/Difference Formulas:

1) Simplify

a) $\cos 3x \cos 5x - \sin 3x \sin 5x$

$= \cos(3x + 5x)$

$= \cos(8x)$

2) Evaluate exactly

a) $\dfrac{\tan\left(\frac{10\pi}{21}\right) - \tan\left(\frac{\pi}{7}\right)}{1 + \tan\left(\frac{10\pi}{21}\right)\tan\left(\frac{\pi}{7}\right)}$

$= \tan\left(\frac{10\pi}{21} - \frac{\pi}{7}\right)$

$= \tan\left(\frac{10\pi}{21} - \frac{3\pi}{21}\right)$

$= \tan\left(\frac{7\pi}{21}\right)$

$= \tan\left(\frac{\pi}{3}\right)$

$= \sqrt{3}$

4) Prove that $\sin\left(\frac{3\pi}{2} - \right.$ equals $-\cos x$

$LS = \sin\left(\frac{3\pi}{2} - x\right)$

$= \sin\frac{3\pi}{2}\cos x - \cos\frac{3\pi}{2}\sin$

$= (-1)\cos x - 0 \sin x$

$= -\cos x$

$RS = -\cos x$

$\therefore LS = RS$

Double Angle Formulas:

2) $\cos(2x)$

$= \cos(x + x)$

$= \cos x \cos x - \sin x \sin x$

$= \cos^2 x - \sin^2 x$ * sub $\cos^2 x = 1 - \sin^2 x$

$= (1 - \sin^2 x) - \sin^2 x$

$\cos 2x = 1 - 2\sin^2 x$

OR

sub $\sin^2 x = 1 - \cos^2 x$

$= \cos^2 x - \sin^2 x$

$= \cos^2 x - (1 - \cos^2 x)$

$= \cos^2 x - 1 + \cos^2 x$

$\cos 2x = 2\cos^2 x - 1$

Practice Questions:

1. **Simplify**

a) $\cos \frac{5\pi}{12}$

2. **Evaluate**

a.) $\cos\left(\frac{-11\pi}{12}\right)$ b.) $\sin \frac{13\pi}{12}$

3. **Prove the following**

a.) $\sec 2x - \tan 2x = \frac{\cos x - \sin x}{\cos x + \sin x}$ b.) $\sin(x+y)\sin(x-y) = \cos^2 y - c$

4. **Solve**

$\sin^2 x - 2\sin x + 1 = 0 \quad -2\pi \le x \le 2\pi$

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Practice Questions

① Simplify:

a) $\cos\frac{5\pi}{12}$

$= \cos\left(\frac{\pi}{6}+\frac{\pi}{4}\right)$

$= \cos\frac{\pi}{6}\cos\frac{\pi}{4} - \sin\frac{\pi}{6}\sin\frac{\pi}{4}$

$= \left(\frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{2}\cdot\frac{1}{\sqrt{2}}\right)$

$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$

$= \frac{\sqrt{3}-1}{2\sqrt{2}}$

$=$

② Evaluate:

a) $\cos\left(\frac{-11\pi}{12}\right) \cdot -1$

$= \cos\left(\frac{11\pi}{12}\right)$

$= \cos\left(\frac{8\pi}{12}+\frac{3\pi}{12}\right)$

$= \cos\left(\frac{2\pi}{3}+\frac{\pi}{4}\right)$

$= \cos\frac{2\pi}{3}\cos\frac{\pi}{4} - \sin\frac{2\pi}{3}\sin\frac{\pi}{4}$

$= \left(\frac{-1}{2}\cdot\frac{1}{\sqrt{2}}\right) - \left(\frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}}\right)$

$= \frac{-1-\sqrt{3}}{2\sqrt{2}}$

b) $\sin\left(\frac{13\pi}{12}\right)$

$= \sin\left(\frac{9\pi}{12}+\frac{4\pi}{12}\right)$

$= \sin\left(\frac{3\pi}{4}+\frac{\pi}{3}\right)$

$= \sin\frac{3\pi}{4}\cos\frac{\pi}{3} + \cos\frac{3\pi}{4}\sin\frac{\pi}{3}$

$= \left(\frac{1}{\sqrt{2}}\cdot\frac{1}{2}\right) + \left(\frac{-1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2}\right)$

$= \frac{1-\sqrt{3}}{2\sqrt{2}}$

③ Prove the following:

a) $\sec 2x - \tan 2x = \frac{\cos x - \sin x}{\cos x + \sin x}$

$LS = \sec 2x - \tan 2x$

$= \frac{1}{\cos 2x} - \frac{\sin 2x}{\cos 2x}$

$= \frac{1 - 2\sin x\cos x}{1 - 2\sin^2 x}$

$RS = \frac{\cos x - \sin x}{\cos x + \sin x} \cdot \frac{\cos x - \sin x}{\cos x - \sin x}$

$= \frac{(\cos x - \sin x)^2}{\cos^2 x - \sin^2 x}$

$= \frac{\cos^2 x - 2\cos x\sin x + \sin^2 x}{1 - \sin^2 x - \sin^2 x}$

$= \frac{1 - 2\cos x\sin x}{1 - 2\sin^2 x}$

$= LS$

$\boxed{\therefore LS = RS}$

Practice Questions Continued

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③ Prove the following:

b) $\sin(x+y)\sin(x-y) = \cos^2 y - \cos^2 x$

$LS = \sin(x+y)\sin(x-y)$

$= (\sin x\cos y + \cos x\sin y)(\sin x\cos y - \cos x\sin y)$

$= \sin^2 x\cos^2 y - \cos^2 x\sin^2 y$

$= (1-\cos^2 x)\cos^2 y - \cos^2 x(1-\cos^2 y)$

$= \cos^2 y - \cos^2 x\cos^2 y - \cos^2 x + \cos^2 x\cos^2 y$

$= \cos^2 y - \cos^2 x$

$RS = \cos^2 y - \cos^2 x$

$\therefore LS = RS$

④ Solve: Given $-2\pi \le x \le 2\pi$

$\sin^2 x - 2\sin x + 1 = 0$

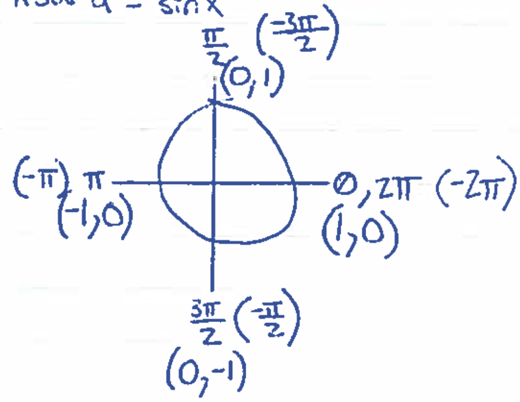
*sub $a = \sin x$

* $a^2 - 2a + 1 = 0$

$(a-1)(a-1) = 0$

$(\sin x - 1)(\sin x - 1) = 0$

$\sin x = 1$



$x = \cancel{\frac{-\pi}{2}}, \frac{-3\pi}{2}, \frac{\pi}{2}, \cancel{\frac{3\pi}{2}}$

$x \neq \frac{-\pi}{2}, \frac{3\pi}{2}$