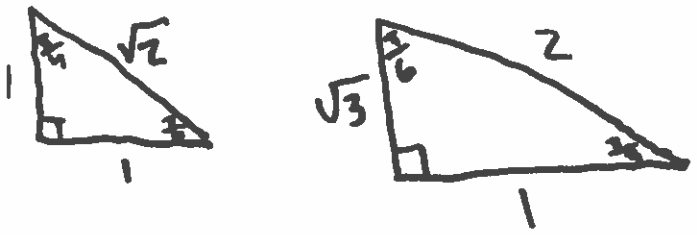
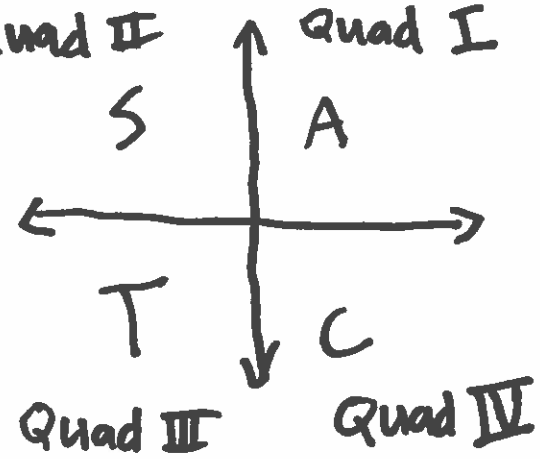
Unit 5: Trig I

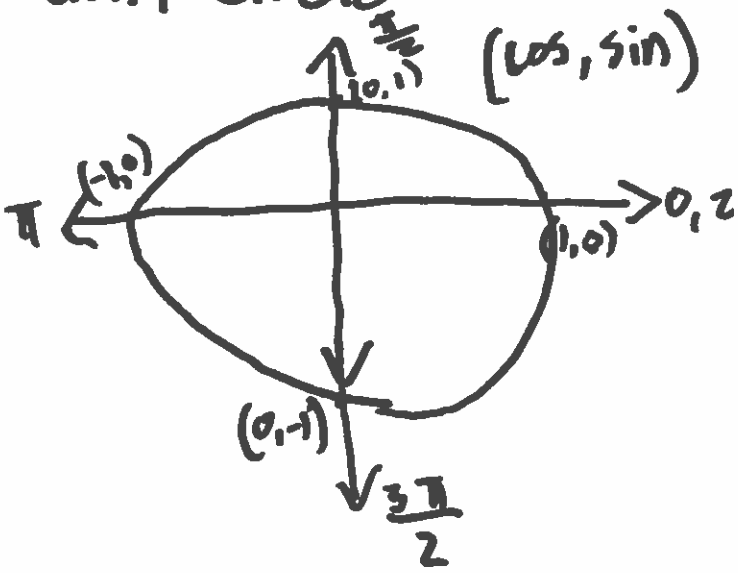
Special Triangles



CAST rule:



Unit circle:



Primary Trig ratios

$\sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$ $\quad \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$\cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$ $\quad \cos \frac{\pi}{3} = \frac{1}{2}$

$\tan \frac{\pi}{4} = 1$ $\quad \tan \frac{\pi}{3} = \sqrt{3}$

$\sin \frac{\pi}{6} = \frac{1}{2}$ $\quad \tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$

$\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$

Conversions:

$1^\circ = \frac{\pi}{180}$ rad

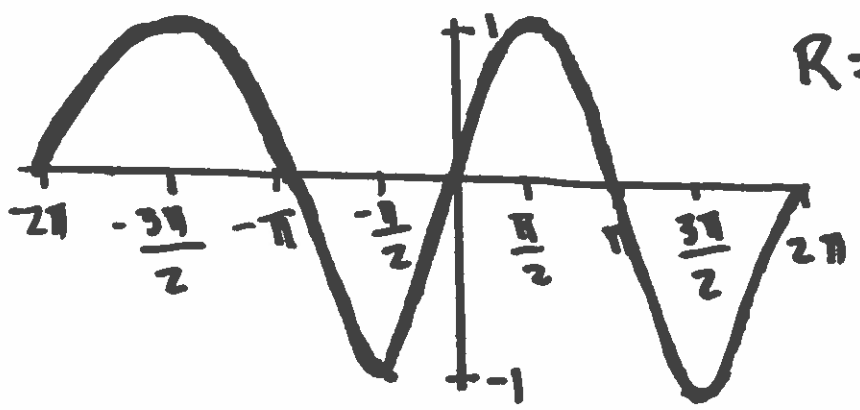
$1 \text{ rad} = \frac{180}{\pi}$

Arc length:

$\theta = \frac{a}{r} = \frac{\text{arc length}}{\text{radius}}$ or $a = \theta r$

Graphing trig functions:

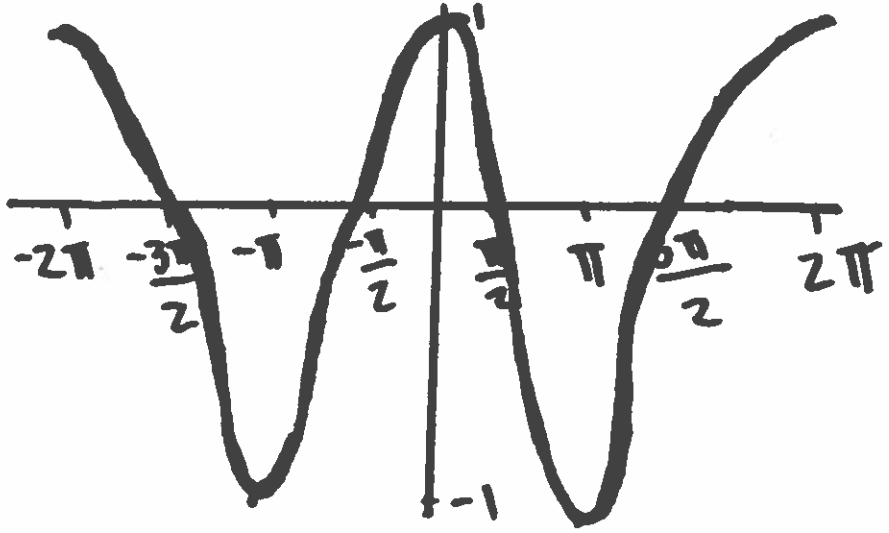
basic sine function:



$D=\{x\in\mathbb{R}\}$

$R=\{y\in\mathbb{R} \mid -1\leq y\leq 1\}$

basic cosine graph:



$D=\{x\in\mathbb{R}\}$ $R=\{y\in\mathbb{R} \mid -1\leq y\leq 1\}$

Trig Transformations:

$$y = a\sin[k(\theta - p)] + q$$

a= vert. stretch|comp (When a is negative, flip on x-axis)
(a>1, stretch or a<1, compression)

k= horizontal stretch|comp.
(k is negative flip on y-axis)
(k>1, comp or k<1, stretch)

p= shift left or right
$\theta + p$ = left
$\theta - p$ = right

q= shift up or down

Practice Questions:

1. Convert rad to deg

a) $\frac{5\pi}{4}$ b) 2.5 c) -3π

2. Convert degree to rad

a) 120° b) -135° c) 450°

3. The London Eye Ferris wheel has a diameter of 135 m and completes one revolution in 30 minutes.

a) Determine the angular velocity in radians per second.

b) How far has a rider travelled 10 minutes into the ride?

4. Given $\cos\theta = \frac{-5}{13}$ and θ is in QIII, determine $\tan\theta$ and $\csc\theta$.

5. Evaluate Exactly

$\tan^2\left(\frac{\pi}{6}\right) - \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}\right)$

6. Graph the functions:

a) $y = 2\sin\left(2\theta - \frac{\pi}{3}\right) + 1$ 0 to 2π

b) $y = \cos\left[\frac{1}{3}\left(\theta + \frac{\pi}{3}\right)\right] + 4$ -2π to 2π

Practice Questions Continued

7. A carnival Ferris wheel with a radius of 7m makes one comple revolution every 2 minutes. Passengers get on at the lowest point which is 1m above the ground.

a) Draw a graph to show a passengers Height over Time.

b) State an equasion for the graph which expresses the Height as a Function of time.

c) Determine the height of a rider above the ground after 15 seconds.

d) At what time will the rider be 11 m above the ground during one revolutio

8. Water at a beach has an average depth of 1M at low tide. Average depth at high tide is 8 m. One cycle takes 12 hours. Low tide is at midnight.

a) Draw a graph to show depth of water over a 24 hour period.

b) state an equasion for the graph using wsine function and another using the sine function.

c) Determine the depth of water at 2:00 am.

d) If the water must be at least 3m to dive in safely, what daylight hours should people dive?

Unit 5- Solutions

1. a) $\frac{5\pi}{4} \times \frac{180}{\pi}$

$= 225°$

b) $2.5 \times \frac{180}{\pi}$

$= \frac{450}{\pi}$

$\doteq 143.2°$

c) $-3\pi \times \frac{180}{\pi}$

$= -540°$

2. a) $120° \times \frac{\pi}{180}$

$= \frac{2\pi}{3}$

b) $-135° \times \frac{\pi}{180}$

$= -\frac{3\pi}{4}$

c) $450° \times \frac{\pi}{180}$

$= \frac{5\pi}{2}$

3. a) $w = \frac{2\pi}{1800}$ ← angle for 1 rev / ← time (s)

$= \frac{\pi}{900}$ rad/sec

b) $a = \theta r$

$= \left(\frac{2\pi}{3}\right)(67.5)$

$\doteq 141.4$ metres

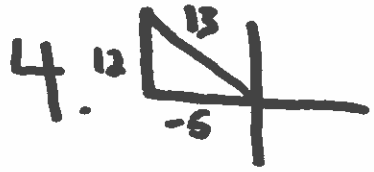
$r = \frac{135}{2}$

$= 67.5$

30 min $= 2\pi$

10 min $= \frac{2\pi}{3}$

4.



$a^2 + b^2 = c^2$

$-5^2 + b^2 = 13^2$

$25 + b^2 = 164$

$b = 12$

$\tan\theta = \frac{12}{-5}$

$\csc\theta = \frac{13}{12}$

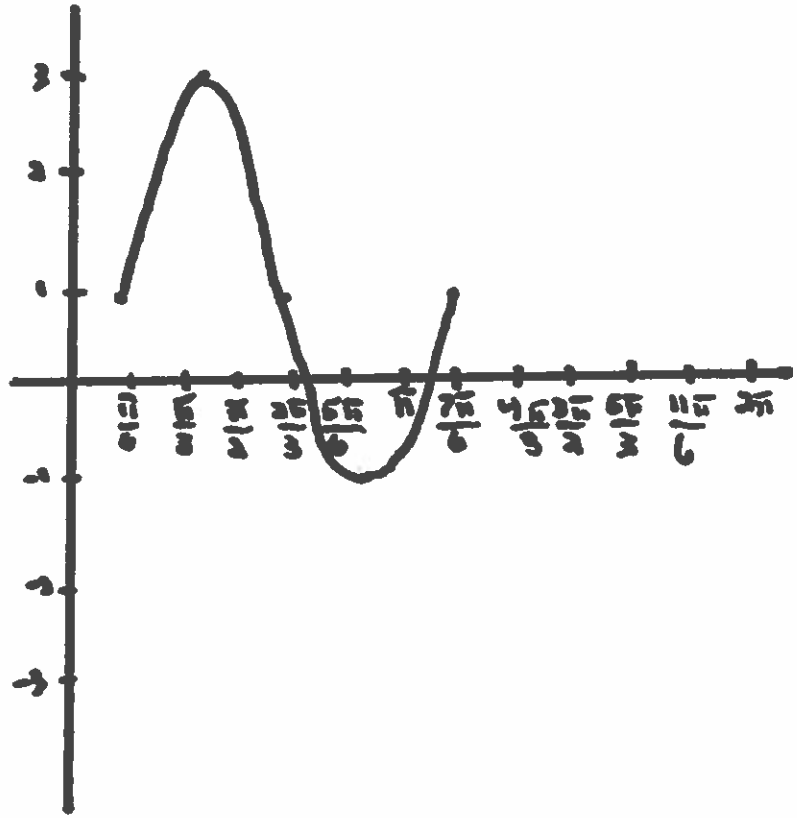
5. $\tan^2\left(\frac{\pi}{6}\right) - \sin\left(\frac{\pi}{4}\right)\left(\frac{\pi}{4}\right)\cos$

$= \left(\frac{1}{\sqrt{3}}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)$

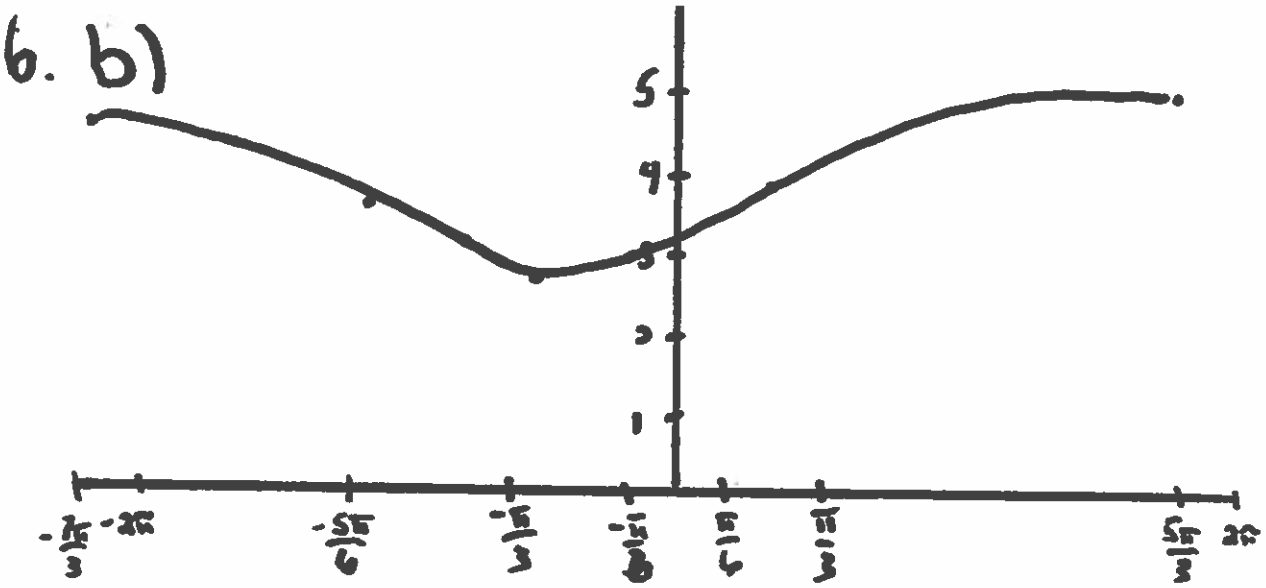
$= \frac{1}{3} - \frac{1}{2}$

$= \frac{-1}{6}$

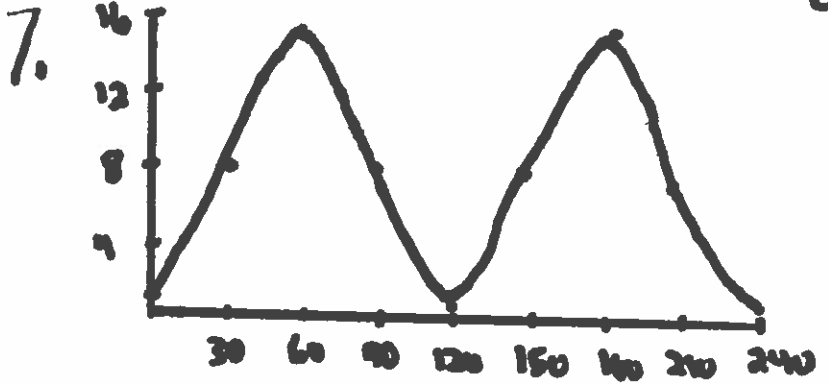
6. a)



6. b)



7. a)



b) $a = \left|\frac{m-n}{2}\right|$

$= \frac{15-1}{2}$

$= 7$

$\frac{2\pi}{k} = 120$

$\frac{\pi}{60} = k$

Period = 120 sec

$q = 8$ (vertical shift up)

cos: $h = -7\cos\left(\frac{\pi}{60}t\right)+8$

sin: $h = 7\sin\left[\frac{\pi}{60}(t-30)\right]+8$

c) sub $t = 15$

$h = -7\cos\left[\frac{\pi}{60}(15)\right]+8$

$= -7\cos\left(\frac{\pi}{4}\right)+8$

$\doteq 3.1$ m

$\therefore$ 3.1 m after 15 sec

d) Sub $h = 11$ m

$11 = -7\cos\left(\frac{\pi}{60}t\right)+8$

$3 = -7\cos\left(\frac{\pi}{60}t\right)$

$\frac{-3}{7} = \cos\left(\frac{\pi}{60}t\right)$

$\frac{\pi}{60}t = \cos^{-1}\left(\frac{3}{7}\right)$

$\frac{\pi}{60}t \doteq 2.01$

$t = 2.01 \times \frac{60}{\pi}$

$t = 38.4$ sec.

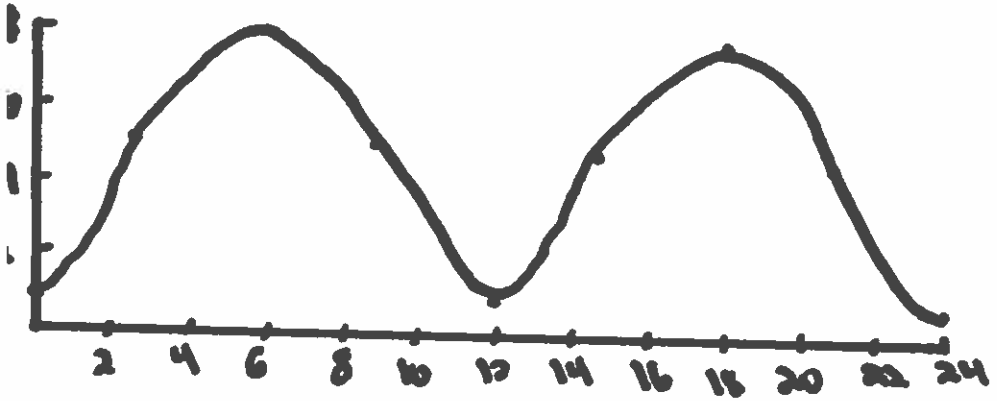
Also $\frac{\pi}{60}t = 2\pi - 2.01$

2nd value for t

$= 120 - 38.4$

$= 81.6$ seconds

8a)



b) $a = \left|\frac{M-m}{2}\right|$

$= \frac{8-1}{2}$

$= 3.5$

$\frac{2\pi}{k} = 12$

$\frac{\pi}{6} = k$

$Q = 4.5$ (shift up 4.5)

Period = 12 hours

Sin: Phase shift right 3

Cos: use $y = -\cos x$ w/ no phase shift

$\therefore$ cos: $d = -3.5\cos\left(\frac{\pi}{6}t\right) + 4.5$

sin: $d = 3.5\sin\left[\frac{\pi}{6}(t-3)\right] + 4.5$

c) Sub $t = 2$

$d = -3.5\cos\left[\frac{\pi}{6}(2)\right] + 4.5$

$= 2.75$ m

d) $d = 3$

$3 = -3.5\cos\left(\frac{\pi}{6}t\right) + 4.5$

$\frac{3-4.5}{-3.5} = \cos\left(\frac{\pi}{6}t\right)$

$1.13 = \frac{\pi}{6}t$

$t = 1.13\left(\frac{\pi}{6}\right)$

$t \doteq 2.15$ hours (2:09 am)

$t_2 = 12 - 2.15$

$= 9.85$ (9:51 am)

$t_3 = 12 + 2.15$

$= 14.15$ (2:09 pm)

$t_4 = 24 - 2.15$

$= 21.85$ (9:51 pm)