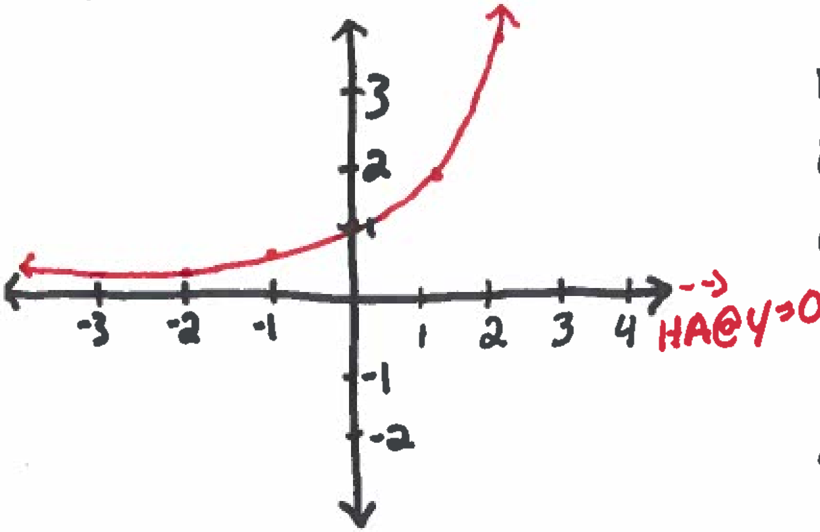
# Exponential Function




**Properties** (if $b>1$)

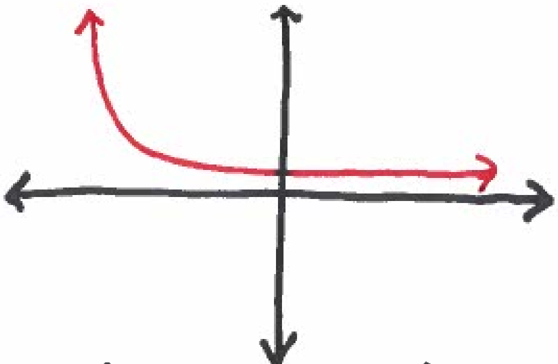
Domain $=\{x \in \mathbb{R}\}$

Range $=\{y \in \mathbb{R} \mid y>0\}$

y int. $=1$ no x int.

$\lim_{x\to-\infty} f(x)=0$ $\lim_{x\to\infty} f(x)=+\infty$

Ex. $y=2^x$ (general form $y=b^x$) ↳ base



**Properties** (if $0<b<1$)

*Note $y=b^x$ not defined if $b<0$

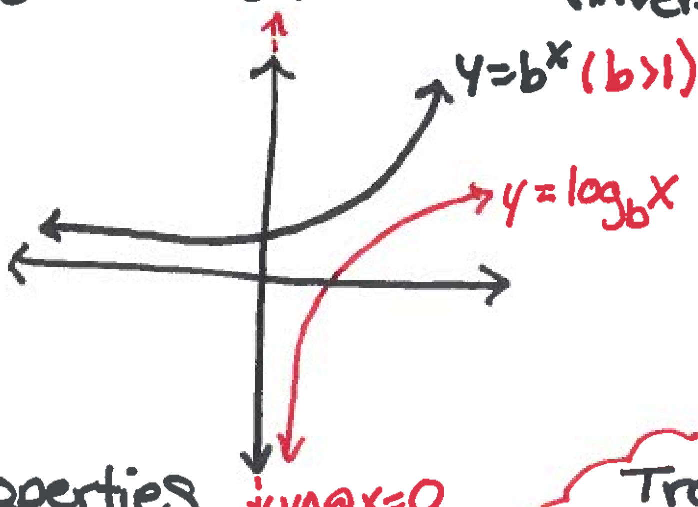
Domain $\{x \in \mathbb{R}\}$ Range $=\{y \in \mathbb{R} \mid y>0\}$

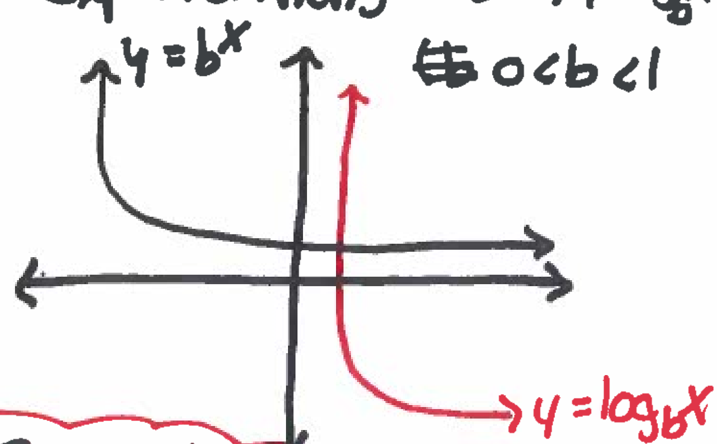
y int. $=1$ no x int.

*opposite limits

# Logarithmic Function

(inverse of exponential) $x=b^y \Rightarrow y=\log_b x$




**Properties**

$D=\{x \in \mathbb{R} \mid x>0\}$

$R=\{y \in \mathbb{R}\}$

X int. $=1$ no y int.

vertical asymptote $x=0$

**Transformations**

$y=-2^{x-2}+1$ right 2, up 1
↳ reflect in x-axis

$y=2^{-x-2}-1$ down 1
↳ reflect in y-axis

*Note: when applying vertical shifts move the asymptote as well

# Exponent Laws and Equations

Recall: Generally $x^{\frac{a}{b}} = (\sqrt[b]{x})^a$ or $x^{a/b} = \sqrt[b]{x^a}$ and $x^{-a} = (\frac{1}{x})^a$

Ex. $\dfrac{(9x)^{3/2}(16x)^{1/4}}{8(x^2)^{1/3}}$

$= \dfrac{(\sqrt{9})^3 x^{3/2} (\sqrt[4]{16}) x^{1/4}}{(\sqrt[3]{8})\, x^{2/3}}$

$= \dfrac{(27x^{3/2})(2x^{1/4})}{2x^{2/3}}$

$= \dfrac{27x^{7/4}}{x^{2/3}}$

$= 27x^{\frac{23}{12}}$

Ex. Solve for x

a. $9^{x+1} = \frac{1}{27}$

$3^{2(x+1)} = 3^{-3}$

$2x+2 = -3$

$2x = -5$

$x = -\frac{5}{2}$

b. $\frac{1}{2}(2^{\sqrt{x}}) = 128$

$2^{\sqrt{x}} = 128(2)$

$2^{\sqrt{x}} = 256$

$2^{\sqrt{x}} = 2^8$

$\sqrt{x} = 8$

$x = 64$

Tips: change to same base
set exponents equal

# Exponential Growth and Decay

## Appreciation/Depreciation

growth $b>1$ $A = A_0(b)^t$ where $b = 1 + \frac{p}{100}$

decay $0<b<1$ $A = A_0(b)^t$ where $b = 1 - \frac{p}{100}$

$A$ = final amount $A_0$ = initial amount
$b$ = growth/decay factor
$t$ = time (# of growth/decay periods)

## Doubling/Half-Life

Doubling: $b=2$ $A = A_0(b)^{\frac{t}{d}}$ Half Life: $b = \frac{1}{2}$ $A = A_0(b)^{\frac{t}{h}}$

$A$ = final amount
$A_0$ = initial amount
$d$ = doubling time
$h$ = given half-life

## Compound Interest

$A = P(1+i)^n$ → # growth periods

$A$ $A_0$ $b = 1 + \frac{p}{100}$

$A$ = amount of investment w/ interest

$P$ = initial amount invested

$i = \dfrac{\text{interest rate}}{\text{interest periods/year}}$

$n$ = number of interest periods

Ex. If a population of bacteria double every $\frac{1}{2}$hr and current population = 500 find:

a. population at 3hrs

$A = 500(2)^{\frac{3}{0.5}}$

$A = 500(2)^6$

$A = 32000$

b. How long it takes to triple
find "t" when $A = 1500$

$1500 = 500(2)^{\frac{t}{0.5}}$

$3 = 2^{\frac{t}{.5}}$

$\log_2 3 = \frac{t}{0.5}$

$0.5\left(\frac{\log 3}{\log 2}\right) = t$

$0.79 = t$

# Evaluating Logarithms

Exponential Form $\Longrightarrow$ Logarithmic Form * y is the exponent to which 'b' is raised to get x

$x=b^y$ $\qquad$ $y=\log_b x$

Example: Log Form

a) $9^2=81$
$\log_9 81=2$

b) $5^0=1$
$\log_5 1=0$

Given $x=b^y$
sub $y=\log_b x$
$x=b^{\log_b x}$

or $y=\log_b x$
sub $x=b^y$
$y=\log_b b^y$

$\Longrightarrow$ Example: Evaluate

a) $5^{\log_5 25}$
$=25$

b) $\log_4(\sqrt{4})^{\frac{1}{2}}$
$=\log_4(4)^{\frac{1}{4}}$
$=\frac{1}{4}$

Example: Solve for x

a) $\log_{10} x=6$
$x=10^6$
$x=1\,000\,000$

b) $\log_4 x=2$
$x=4^2$
$x=16$

## Log Properties & Change of Base

1) $\log_b(m\cdot n)=\log_b m+\log_b n$
2) $\log_b\left(\frac{m}{n}\right)=\log_b m-\log_b n$
3) $\log_b(m^n)=n\cdot\log_b m$

$\Longrightarrow$ Example: Use Log Properties

a) $\log_2\left(\frac{16x}{y^4}\right)$
$=\log_2(16x)-\log_2(y^4)$
$=\log_2 16+\log_2 x-4\cdot\log_2 y$
$=4+\log_2 x-4\cdot\log_2 y$

Example: Simplify to single Logarithm

a) $3\cdot\log_2 x+\log_2 y$
$=\log_2 x^3+\log_2 y$
$=\log_2(x^3 y)$

Example: Evaluate

a) $\log_2 112-\log_2 7$
$=\log_2\left(\frac{112}{7}\right)$
$=\log_2 16$
$=4$

## Change of Base Formula

$$\log_b x=\frac{\log_a x}{\log_a b}$$

Proof: Given $y=\log_b x \Longleftrightarrow x=b^y$

Take "log" of both sides $\qquad \log x=\log b^y$

Apply log properties $\qquad \log x=y\cdot\log b$

Rearrange for "y" $\qquad y=\dfrac{\log x}{\log y}$

Example

a) $\log_4 5$
$=\dfrac{\log 5}{\log 4}$
$\approx 1.16$

b) $\log_3 4.5$
$=\dfrac{\log 4.5}{\log 3}$
$=1.37$

## Solving Exponential + Logarithmic Equations

Strategies

write both sides with same base

or

take the "log" of both sides! $x = b^y \Rightarrow y = \log_b x$

Ex. Solve

a) $12.5 = 200(\frac{1}{2})^x$

divide by 200

$\frac{1}{16} = (\frac{1}{2})^x$

$(\frac{1}{2})^4 = (\frac{1}{2})^x$

$4 = x$

## Log Equations: more strategies

- simplify using log properties
- take exponent of both sides
- check that solutions are within given domain
- rewrite in exponential form

Example: Solve

a) $\log_x 0.04 = -2$

write in exponential form

$x = \sqrt{\frac{1}{0.04}}$

$x = \frac{1}{0.2}$ not negative since $x > 0$

$x = 5$

## Sample Questions

1) Graph a. $y = -2^{x+3} + 2$ b. $y = \log_2(x-2)+$

2) Evaluate and write in exponential form

a. $\log_2(\frac{1}{16}) = -4$ b. $\log_3(3\sqrt{3}) = \frac{3}{2}$

3) Solve for x

a. $5^{x+3} = 30$ b. $2^{x+2} - 2^x = 24$

c. $\log_8(2-x) + \log_8(4-x) = 1$

4) Expand using log properties

a. $\log_5 \sqrt{x^3 y^4}$

5) Simplify to a single logarithm

a. $2\log_5 x - \frac{1}{2} \cdot \log_5(x+1) + 3\log_5 z$

6) Evaluate using log properties

a. $\log_6 \sqrt{\frac{1}{6}}$

7) Radium has a half-life of 1690 a. If 90 mg are currently present...

a. How much will there be 100 years from now

b. when will there be 72 mg remaining?

8) An atomic bomb measures 6.3 on the RS. An Earthquake measured 7.1. How much more intense was the earthquake than the bomb

Unit 4: Sample Question Answers June $10^{th}$ 2015

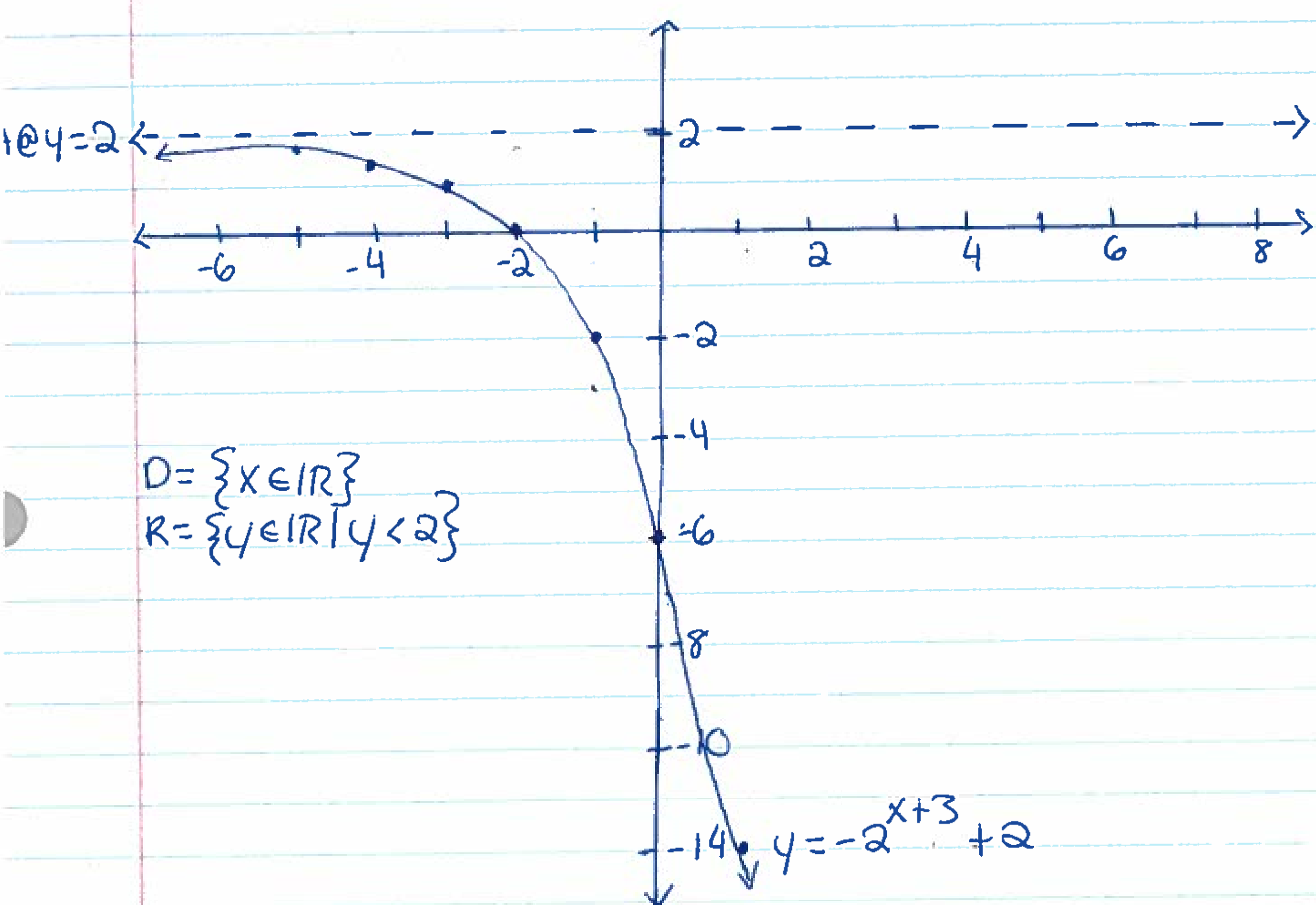
1) Graph

a. $y=-2^{x+3}+2$

Base: $y=2^x$

Reflection in x-axis, left 3 units, up 2 units

HA @ $y=2$



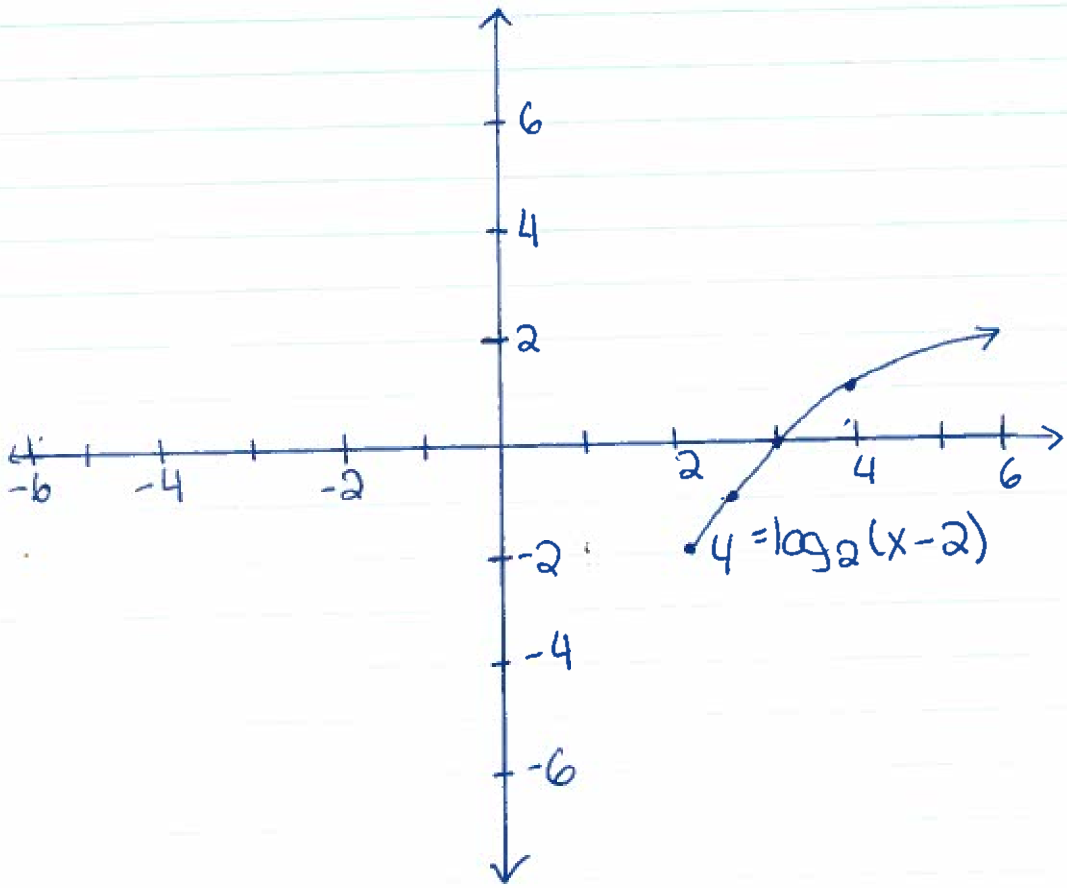

$D=\{x \in \mathbb{R}\}$

$R=\{y \in \mathbb{R} \mid y<2\}$

b. $y = \log_2(x-2)$

Base: $y = \log_2 x \Rightarrow$ inverse $y = 2^x$

Shift right 2 units



2) Evaluate and write in exponential form

a. $\log_2\left(\frac{1}{16}\right) = -4$

$$2^{-4} = \frac{1}{16}$$

b. $\log_3\left(3\sqrt{3}\right) = \frac{3}{2}$

$$3^{\frac{3}{2}} = 3\sqrt{3}$$

3) Solve for X

a. $5^{x+3} = 30$

$$\log 5^{x+3} = \log 30$$

$$x+3 = \frac{\log 30}{\log 5}$$

$$x = \frac{\log 30}{\log 5} - 3$$

$$x = -0.89$$

b. $2^{x+2} - 2^{x} = 24$

$$(2^x \cdot 2^2) - 2^x = 24$$

$$2^x(2^2 - 1) = 24$$

$$2^x(3) = 24$$

$$2^x = 8$$

$$2^x = 2^3$$

$$x = 3$$

c. $\log_8 \underbrace{(2-x)}_{x \text{ must be } < 2} + \log_8 \underbrace{(4-x)}_{x < 4} = 1$ combine with log property

$$\log_8[(2-x)(4-x)] = 1$$

$$\log_8(8 - 6x + x^2) = 1$$

$$8 = 8 - 6x + x^2$$

$$0 = x^2 - 6x$$

$$0 = x(x-6)$$

$0 = x$ or $x = 6 \rightarrow$ inadmissible because $x < 2$

4) Expand using log properties

a. $\log_5 \sqrt{x^3 y^9}$

$$= \log_5 (x^3 y^9)^{\frac{1}{2}}$$

$$= \frac{1}{2} \log_5 (x^3 y^9)$$

$$= \frac{1}{2}[\log_5 x^3 + \log_5 y^9]$$

$$= \frac{1}{2}[3\log_5 x + 9\log_5 y]$$

5) Simplify to a single logarithm

a. $2\log_5 x - \frac{1}{2}\log_5(x+1) + 3\log_5 z$

$= \log_5 x^2 - \log_5 \sqrt{x+1} + \log_5 z^3$

$= \log_5 \frac{x^2 z^3}{\sqrt{x+1}}$

6) Evaluate using log properties

a. $\log_6 \sqrt{\frac{1}{6}}$

$= \log_6 \left(\frac{1}{6}\right)^{\frac{1}{2}}$

$= \frac{1}{2}\log_6\left(\frac{1}{6}\right)$

$= \frac{1}{2}\log_6(6^{-1})$

$= -\frac{1}{2}$

7) a. $b = \frac{1}{2}$

$A = 90\text{mg}$

$A_0 = ?$

$\frac{t}{h} = \frac{100}{1690}$

$A = A_0(b)^{\frac{t}{h}}$

$= 90\left(\frac{1}{2}\right)^{\frac{100}{1690}}$

$= 86.4\text{ mg}$

b. $72 = 90\left(\frac{1}{2}\right)^{\frac{t}{1690}}$

$\frac{72}{90} = \left(\frac{1}{2}\right)^{\frac{t}{1690}}$

$0.8 = \left(\frac{1}{2}\right)^{\frac{t}{1690}}$

$\log_{0.5} 0.8 = \frac{t}{1690}$

$\frac{\log 0.8}{\log 0.5} = \frac{t}{1690}$

$t = (1690)\left(\frac{\log 0.8}{\log 0.5}\right)$

$t = 544$ years

8) $\log\left(\frac{I_1}{I_2}\right) = 7.1 - 6.3$

$= 0.8$

$\frac{I_1}{I_2} = 10^{0.8}$

$= 6.3$

or

$\frac{10^{7.1}}{10^{6.3}}$

$= 10^{0.8}$

$= 6.3$