

UNIT 5. RATIONAL FUNCTIONS

RATIONAL FUNCTION: A RATIONAL FUNCTION HAS THE FORM $h(x) = \frac{f(x)}{g(x)}$, WHERE $f(x)$ AND $g(x)$ ARE POLYNOMIALS

THE DOMAIN OF A RATIONAL FUNCTION CONSISTS OF ALL REAL NUMBERS FOR x , EXCEPT THE RESTRICTIONS ON THE DENOMINATOR, SINCE $g(x) \neq 0$

EX. FIND THE DOMAIN, x AND y -INTERCEPT

$$a) f(x) = \frac{7}{x+2}$$

$$D = \{x \in \mathbb{R} \mid x \neq -2\}$$

$$\text{x-INT: } y=0$$

$$\frac{0}{1} = \frac{7}{x+2}$$

$$0 = 7 \text{ (IMPOSSIBLE)} \\ \therefore \text{NO x-INT}$$

$$\text{y-INT: } x=0$$

$$f(x) = \frac{7}{0+2}$$

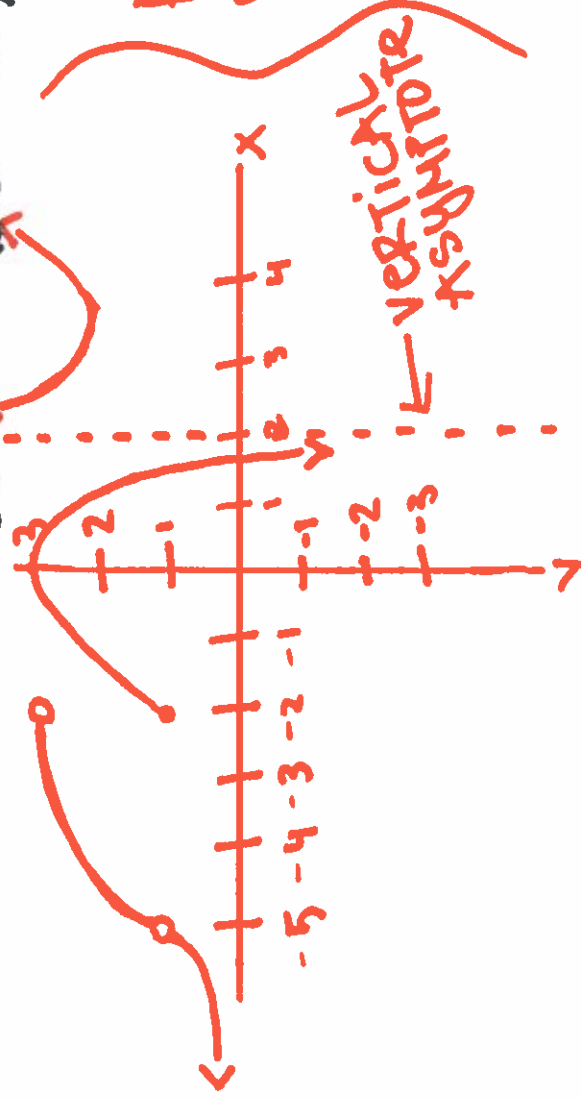
$$= \frac{7}{2} \\ \therefore (0, \frac{7}{2})$$

LIMITS

A LIMIT DESCRIBES WHAT IS HAPPENING TO y , NEAR A SPECIFIC VALUE OF x ALONG A GRAPH OR CURVE (i.e. WHAT NUMBER y IS APPROACHING)

LIMITS CAN APPROACH THE x -VALUE ALONG THE GRAPH FROM THE LEFT (-) OR FROM THE RIGHT (+). IN ORDER FOR A LIMIT TO EXIST, THE LEFT AND RIGHT LIMIT MUST BE EQUAL.

Ex. USING THE GRAPH BELOW, FIND THE FOLLOWING LIMIT:



VERTICAL ASYMPTOTES

RATIONAL FUNCTIONZ USUALLY HAVE RESTRICTIONS ON THEM THAT CREATE VERTICAL ASYMPTOTES \leftrightarrow

VERTICAL ASYMPTOTES (VA) ARE LINES A CURVE APPROACHES, BUT NEVER TOUCHES. THEY ARE CREATED WHEN THE LIMITS ON EITHER SIDE OF THE RESTRICTION. APPROACH ∞ , $-\infty$ OR $\pm \infty$.

EX. FIND THE X AND Y-INT AND THE VA OF THE FOLLOWING:

$$f(x) = \frac{1}{x+1}$$

X-INT
 $0 = \frac{1}{x+1}$

$0 = 1$
 \therefore NO X-INT


Y-INT
 $f(x) = \frac{1}{0+1}$

$y = 1$
 $\therefore (0, 1)$

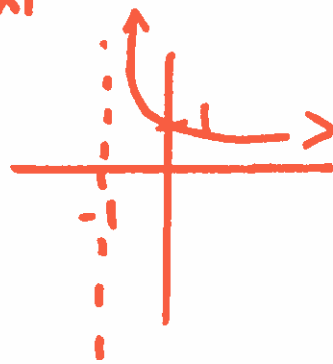
$$D = \{x \in \mathbb{R} \mid x \neq -1\}$$

CHECK VA AT $x = -1$

$\lim_{x \rightarrow -1^-} f(x)$
 $= \frac{1}{-small}$
 $= -\infty$



$\lim_{x \rightarrow -1^+} f(x)$
 $= \frac{1}{+small}$
 $= \infty$
 \therefore VA @ $x = -1$



Practice

Given $f(x) = \frac{x}{x(x-1)^2}$ determine the following:

- a) Domain + Range
- b) X + Y Intercepts
- c) Symmetry
- d) Vertical + Horizontal Asymptotes
- e) Graph the function