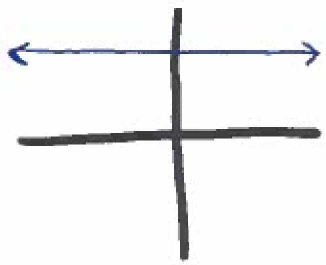
Unit 2: Polynomials II

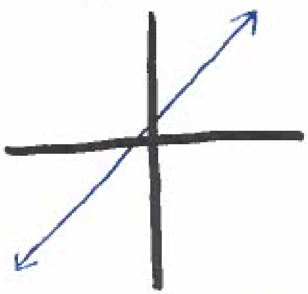
Types of Polynomial Functions:

1) Constant Function



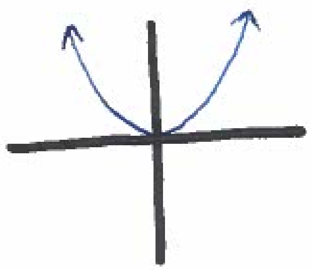
→ Degree: Ø
→ Eg: $f(x) = a$
*Horizontal Line

2) Linear Function



→ Degree: 1
→ Eg: $f(x) = ax + a_1$
*straight Line

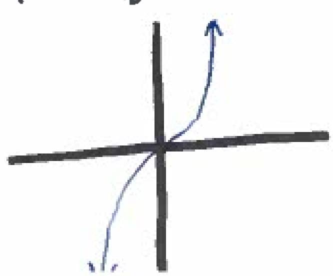
3) Quadratic Function



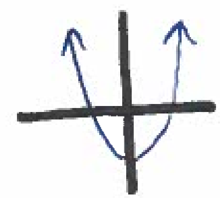
→ Degree: 2
→ Eg: $f(x) = ax^2 + a_1x + a_2$

4) Cubic Function

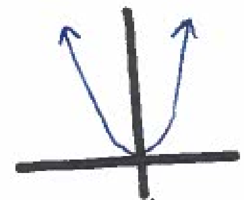
→ Degree: 3
→ Eg: $f(x) = ax^3 + a_1x^2 + a_2x + a_3$



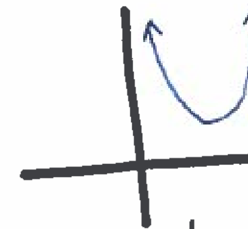
Types of Roots:



2 real distinct roots



2 real equal roots



2 complex roots

End Behaviours:

	+	−
Even Deg.	High - High	Low - Low
Odd Deg.	Low - High	High - Low

Determing Roots w/ Equation:

$f(x) = a(\quad)^1(\quad)^{2/4/6\ldots}(\quad)^{3/5/7\ldots}$

cut (1 distinct)
bounce (_ equal)
lane change

5) Quartic Function (Degree: 4)
↳ $f(x) = ax^4 + a_1x^3 + a_2x^2 + a_3x + a_4$

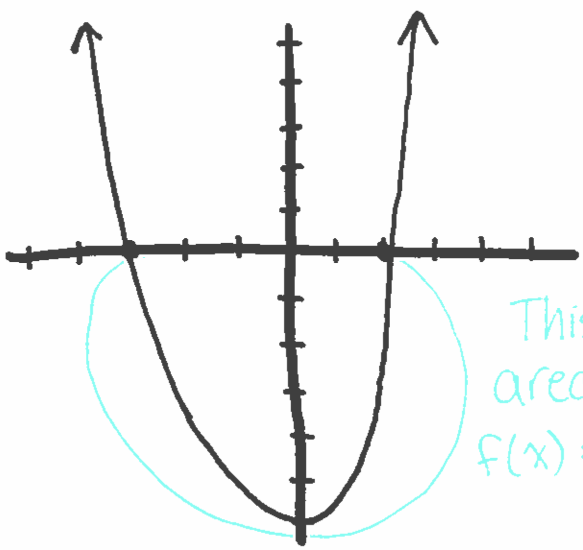
6) Quintic Function (Degree: 5)
↳ $f(x) = ax^5 + a_1x^4 + a_2x^3 + a_3x^2 + a_4x + a$

Solving Polynomial Inequalities

Example: $x^2 + x - 6 \le 0$

Solve Graphically:

$$x^2 + x - 6 \le 0$$
$$(x+3)(x-2) \le 0$$

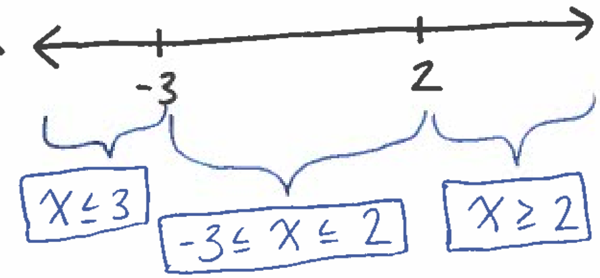


$\therefore x^2 + x - 6 \le$ when

$$-3 \le x \le 2$$

Solve Algebraically:

$$\left.\begin{array}{l} x^2 + x - 6 \le 0 \\ (x+3)(x-2) \le 0 \end{array}\right\}$$



Set up a Chart

	$(x+3)$	$(x-2)$	$f(x)$
$x < -3$	−	−	+
$-3 < x < 2$	+	−	−
$x > 2$	+	+	+

$\therefore x^2 + x - 6 \le 0$ when

$$-3 \le x \le 2$$

Polynomials from Finite Differences

Finite Differences → Differences between consecutive y-values in a table (or consecutive differences)

*Values must be in order with no values missing

Eg1 Determine the 1st Differences

x	$y=f(x)$	$\Delta f(x)$
1	5	+3
2	8	+3
3	11	+3
4	14	+3
5	17	

When the 1st diff's are equal, $f(x)$ is linear

$y=mx+b$ $\quad b=2$

$5=3(1)+b$ $\quad \therefore y=3x+2$

Eg2 Determine 1st & 2nd differences

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$
1	6	+3	+2
2	9	+5	+2
3	14	+7	+2
4	21	+9	
5	30		

When the 2nd diff's are equal, $f(x)$ is quadratic

$y=ax^2+bx+c$

$6=a(1)^2+b(1)+c$

① $\boxed{6=a+b+c}$

$9=a(2)^2+b(2)+c$

② $\boxed{9=4a+2b+c}$

$14=a(3)^2+b(3)+c$

③ $\boxed{14=9a+3b+c}$

② $9=4a+2b+c$

−① $6=a+b+c$

④ $\boxed{3=3a+b}$

③ $14=9a+3b+c$

−② $9=4a+2b+c$

⑤ $\boxed{5=5a+b}$

⑤ $5=5a+b$

−④ $3=3a+b$

$2=2a$

$1=a$

sub $a=1$ in ④

$3=3(1)+b$

$0=b$

sub $b=0, a=1$ in ①

$6=1+0+c$

$5=c$

$\therefore y=x^2+5$

Practice Questions

1. for each of the following state the degree, the number and type of roots :

a) $f(x) = -(x+1)(x^2+3)$ b) $f(x) = x^4 - 3x^2 - 4$ c) $f(x) = x^2(1-x)(x+4)$

2. Provide a sketch of the graphs in #1, include x and y intercepts.

3. Write an equation for each polynomial function below:

a)

X	Y
1	-4
2	-9
3	-20
4	-37
5	-60

b)

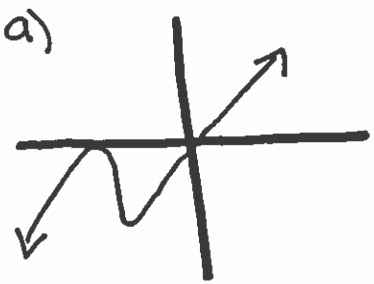
X	Y
0	-1
1	3
2	13
3	41
4	99
5	199

4. Solve the following Polynomial inequality graphically: $x^3 + 7x^2 + 10x \leq 0$

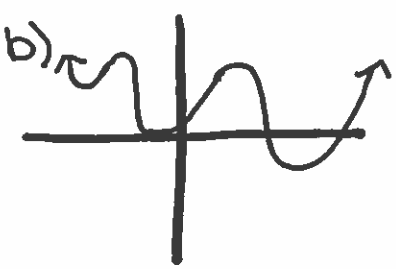
5. Solve the following Polynomial inequality algebraically: $x^4 + 9x^3 + 21x^2 - x - 30 > 0$

6. Predict the degree and describe the roots of:

a)



b)



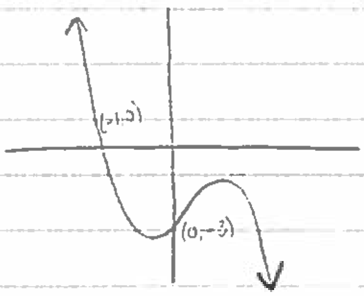
Unit 2: Polynomials II Answers

1) a) Degree: 3
Roots: 1 distinct real
1 complex real

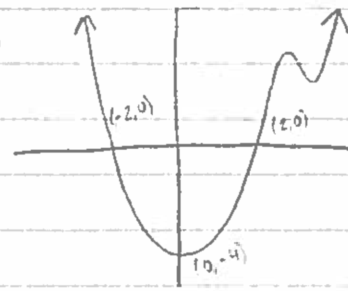
b) Degree: 4
Roots: 2 distinct real
2 complex real

c) Degree: 4
Roots: 2 distinct real
2 equal real

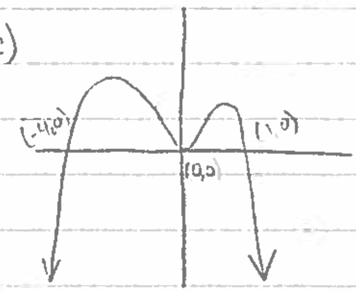
2) a)



b)



c)



3a)

x	f(x)	Δf(x)	Δ²f(x)
1	-4		
		-5	
2	-9		-6
		-11	
3	-20		-6
		-17	
4	-37		-6
		-23	
5	-60		

sub $x=1$, $y=-4$
$-4 = a(1)^2 + b(1) + c$
$-4 = a + b + c$ ①

sub $x=2$, $y=-9$
$-9 = a(2)^2 + b(2) + c$
$-9 = 4a + 2b + c$ ②

sub $x=3$, $y=-20$
$-20 = a(3)^2 + b(3) + c$
$-20 = 9a + 3b + c$ ③

② $-9 = 4a + 2b + c$
① $-4 = a + b + c$
④ $-5 = 3a + b$

③ $-20 = 9a + 3b + c$
② $-9 = 4a + 2b + c$
⑤ $-11 = 5a + b$

⑤ $-11 = 5a + b$
④ $-5 = 3a + b$
$-6 = 2a$
$\boxed{-3 = a}$

sub $a = -3$ into ④
$-5 = 3(-3) + b$
$-5 = -9 + b$
$\boxed{4 = b}$

sub $a = -3$, $b = 4$ into ①
$-4 = -3 + 4 + c$
$-4 = 1 + c$
$\boxed{-5 = c}$

$\therefore f(x) = -3x^2 + 4x - 5$

6a) Degree: 3
Roots: 2 equal real
1 distinct real

b) Degree: 6
Roots: 2 equal real
2 distinct real
2 complex real

b)

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
0	-1			
		4		
1	3		6	
		10		12
2	13		18	
		28		12
3	41		30	
		58		12
4	99		42	
		100		
5	199			

sub $x=1$, $y=3$

$3 = a(1)^3 + b(1)^2 + c(1) - 1$

$4 = a + b + c$

sub $x = 2$, $y = 13$

$13 = a(2)^3 + b(2)^2 + c(2) - 1$

$14 = 8a + 4b + 2c$

sub $x = 3$, $y = 41$

$41 = a(3)^3 + b(3)^2 + c(3) - 1$

$42 = 27a + 9b + 3c$

$14 = 8a + 4b + 2c$

$8 = 2a + 2b + 2c$

$6 = 6a + 2b$

$84 = 54a + 18b + 6c$

$42 = 24a + 12b + 6c$

$42 = 30a + 6b$

$42 = 27a + 9b + 3c$

$12 = 3a + 3b + 3c$

$30 = 24a + 6b$

$42 = 30a + 6b$

$18 = 18a + 6b$

$24 = 12a$

$\boxed{2 = a}$

sub $a = 2$ into

$6 = 6(2) + 2b$

$-6 = 2b$

$\boxed{-3 = b}$

sub $a = 2$, $b = -3$ into

$4 = 2 - 3 + c$

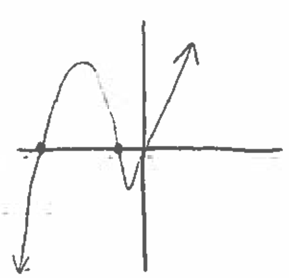
$\boxed{5 = c}$

$\therefore f(x) = 2x^3 - 3x^2 + 5x - 1$

4) $x^3 + 7x^2 + 10x \leq 0$

$x(x^2 + 7x + 10) \leq 0$

$x(x + 5)(x + 2) \leq 0$



$f(x) \leq 0$ when

$x \leq -5 \cup -2 \leq x \leq 0$