MHF 4U1

Problem Set 26 - Solutions

1. a) $A = 40\,000$
$d = .25$
$a_0 = ?$
$t = 1.25$
$b = 2$

$$A = a_0 (b)^{t/d}$$
$$40\,000 = a_0 (2)^{\frac{1.25}{.25}}$$
$$40\,000 = a_0 (2)^5$$
$$\frac{40000}{(2)^5} = a_0$$
$$a_0 = 1250$$

$\therefore$ The initial size of the culture was 1250 bacteria

b) $t = 3$ h
$a_0 = 1250$
$b = 2$
$d = .25$

$$A = a_0 (b)^{t/d}$$
$$A = 1250(2)^{3/.25}$$
$$= 1250(2)^{12}$$
$$= 5\,120\,000$$

$\therefore$ After 3 hours, the size of the culture is 5 120 000 bacteria.

2. $a_0 = 3000$

$A = 48\,000$
$t = 3$
$b = 2$

$$A = a_0 (b)^{t/d}$$
$$48\,000 = 3000 (2)^{3/d}$$
$$16 = (2)^{3/d}$$
$$\frac{3}{d} = \log_2 16$$
$$\frac{3}{d} = 4$$
$$3 = 4d$$
$$\frac{3}{4} = d$$

$\therefore$ The doubling period is $\frac{3}{4}$ hour.

3. $a_0 = 6000$

$A = 33\,600$
$t = 0.5$
$b = 2$

$A = a_0(b)^{t/d}$

$33600 = 6000\,(2)^{0.5/d}$

$5.6 = (2)^{0.5/d}$

$\frac{0.5}{d} = \log_2 5.6$

$\frac{0.5}{d} = \frac{\log 5.6}{\log 2}$

$d(\log 5.6) = 0.5(\log 2)$

$d = \frac{0.5(\log 2)}{\log 5.6}$

$d \doteq 0.201 \xrightarrow[\times 60]{} 12$ minutes

∴ It takes approx. 12 minutes for the number of bacteria to double.

4. $b = 2$
$d = 1$
Let $A = 100a_0$
($100\times$ original amount)

$A = a_0(b)^{t/d}$

$100a_0 = a_0(2)^{t/1}$

$100 = 2^t$

$t = \log_2 100$

$t = \frac{\log 100}{\log 2}$

$t = 6.64$ hours $\Rightarrow$ 6 hrs, 39 min.

$.64 \times 60$

∴ It will take approx. 6 hrs 39 min. for the number of bacteria to be 100 times the original amount.

5. $b = 1 + .02$
$= 1.02$

$a_0 = .65$
$A = 1$

$A = a_0(b)^t$

$1 = .65(1.02)^t$

$\frac{1}{.65} = (1.02)^t$

$t = \log_{1.02}\left(\frac{1}{.65}\right)$

$= \frac{\log\left(\frac{1}{.65}\right)}{\log 1.02}$

$t = 21.75$

$\therefore$ It will cost \$1/L after 21.75 years.

6. $b = \frac{1}{2} = 0.5$

$A = \underbrace{.05 a_0}$
if 95% has decomposed, then 5% of the original amount remains

$h = 30$ seconds

$A = a_0(b)^{t/h}$

$.05 a_0 = a_0 (0.5)^{t/30}$

$.05 = (0.5)^{t/30}$

$\frac{t}{30} = \log_{.5}(.05)$

$t = \frac{\log .05}{\log .5} \times 30$

$t \doteq 129.66$

$\therefore$ It will take approx. 129.66 seconds for 95% of radium-221 to decompose

7. $A = 20mg$

$a_0 = 25$ mg

$b = \frac{1}{2} = 0.5$

$t = 48$

$A = a_0 (b)^{t/h}$

$20 = 25 (0.5)^{48/h}$

$\frac{20}{25} = (0.5)^{48/h}$

$0.8 = (0.5)^{48/h}$

$\frac{48}{h} = \log_{0.5} 0.8$

$= \frac{\log 0.8}{\log 0.5}$

$h = \frac{48(\log 0.5)}{\log 0.8}$

$.10 \times 60$

$h = 149.10 \implies 149$ hrs 6 min

$\therefore$ The half-life is approx. 149 hrs 6 min.

8. $A = 3700$
$P = 3000$
$i = \frac{.0925}{2}$
$= .04625$
$n = ?$

$A = P(1+i)^n$

$3700 = 3000 (1 + .04625)^n$

$\frac{3700}{3000} = (1 + .04625)^n$

$n = \log_{1.04625}\left(\frac{3700}{3000}\right)$

$n = \frac{\log\left(\frac{3700}{3000}\right)}{\log 1.04625}$

$\doteq 4.639$ growth periods

Since there are 2 growth periods every year,
time $= \frac{4.639}{2} \doteq 2.32$ years.

9. $P = 1000$
$A = 1463.44$

$n = 4 \times 2 = 8$

$$A = P(1+i)^n$$

$$1463.44 = 1000(1+i)^8$$

$$\frac{1463.44}{1000} = (1+i)^8$$

$$1+i = \left(\frac{1463.44}{1000}\right)^{1/8}$$

$$1+i \doteq 1.0487$$
$$i \doteq 1.0487 - 1$$
$$i \doteq 0.0487$$

Since $i = .0487$ is the interest rate per half-year, then the annual rate is $.0487 \times 2$ or approx. 9.75% /a.

10. San Francisco earthquake : 8.3
Japan earthquake: 4.8

$$\text{increase in intensity} = \frac{10^{8.3}}{10^{4.8}} = 10^{3.5}$$
$$\doteq 3162$$

∴ The San Francisco earthquake is approx 3162 times more intense than the Japan earthquake.

11. power mower $\Rightarrow$ 106 dB

traffic $\Rightarrow$ 70 dB

$$\text{Increase in intensity} = 10^{\left(\frac{106-70}{10}\right)}$$
$$= 10^{3.6}$$
$$\doteq 3981$$

∴ The power mower is approx 3981 times louder than ordinary traffic

12. noise level before $\Rightarrow$ 79 dB

noise level after $\Rightarrow$ 68 dB

$$\text{decrease in intensity} = 10^{\left(\frac{79-68}{10}\right)}$$
$$= 10^{1.1}$$
$$\doteq 12.6$$

$\therefore$ The noise intensity decreased by a factor of 12.6. (almost 13).