Pre-Exam Assignment - Part 1 $\quad \overline{54}$

1. a) $2x^4+4x+4=x^3+9x^2$

$2x^4-x^3-9x^2+4x+4=0$

$f(1)=2(1)^4-(1)^3-9(1)^2+4(1)+4$
$=0$ ✓
$\therefore (x-1)$ is a factor

$$\begin{array}{c|ccccc} 1 & 2 & -1 & -9 & 4 & 4 \\ & & 2 & 1 & -8 & -4 \\ \hline & 2 & 1 & -8 & -4 & 0 \end{array}$$ ✓

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$(x-1)(2x^3+x^2-8x-4)=0$
$(x-1)[x^2(2x+1)-4(2x+1)]=0$ ✓
$(x-1)(2x+1)(x^2-4)=0$
$(x-1)(2x+1)(x+2)(x-2)=0$ ✓

$x=1 \quad x=-\frac{1}{2} \quad x=-2 \quad x=2$ ✓

b) $2|5-3x|\leq 28$
$|5-3x|\leq 14$ ✓

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If $5-3x\geq 0 \Rightarrow x\leq\frac{5}{3}$

$5-3x\leq 14$
$-3x\leq 9$
$x\geq -3$ ✓

If $5-3x<0 \Rightarrow x>\frac{5}{3}$ ✓

$-(5-3x)\leq 14$
$-5+3x\leq 14$
$3x\leq 19$
$x\leq\frac{19}{3}$ ✓

$\therefore \{x\in\mathbb{R}\,|\,-3\leq x\leq\frac{19}{3}\}$

c) $4^{5x}=16^{2x-1}$

$4^{5x}=(4^2)^{2x-1}$
$4^{5x}=4^{4x-2}$ ✓

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$\therefore 5x=4x-2$
$x=-2$ ✓

d) $\log_3(x^2+2x)=\log_3 x+2$

$\log_3(x^2+2x)-\log_3 x=2$

$\log_3\left[\frac{x^2+2x}{x}\right]=2$ $\quad$ 3

$\log_3(x+2)=2$ ✓

$3^2=x+2$ ✓
$9=x+2$
$x=7$ ✓

e) $\frac{2x(x+6)}{(x-4)} > 0$ zeroes + und. values: $x=0, x=-6, x=4$

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	$2x$	$x+6$	$x-4$	$f(x)$
$x<-6$	$-$	$-$	$-$	$-$
$-6<x<0$	$-$	$+$	$-$	$+$
$0<x<4$	$+$	$+$	$-$	$-$
$x>4$	$+$	$+$	$+$	$+$

$\therefore \{x \in \mathbb{R} \mid -6<x<0 \cup x>4\}$

2. Let $f(x) = kx(x+1)(x-2)$

Given the remainder is 3 when divided by $(x-1)$,

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$$f(1) = k(1)(1+1)(1-2) = 3$$
$$-2k = 3$$
$$k = \frac{3}{-2}$$

$$\therefore f(x) = -\frac{3}{2}x(x+1)(x-2)$$

3. $f(x) = \frac{3x+2}{x-4}$ x-int: $0 = \frac{3x+2}{x-4}$ y-int: $y = \frac{3(0)+2}{(0)-4}$

$D = \{x \in \mathbb{R} \mid x \neq 4\}$

$R = \{y \in \mathbb{R} \mid y \neq 3\}$

$0 = 3x+2$

$x = \frac{-2}{3}$

x-int $\left(-\frac{2}{3}, 0\right)$

$y = -\frac{1}{2}$

y-int $\left(0, -\frac{1}{2}\right)$

V. Asymptotes

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$$\left.\begin{aligned} \lim_{x\to 4^-} \frac{3x+2}{x-4} &\approx \frac{14}{-sm} = -\infty \\ \lim_{x\to 4^+} \frac{3x+2}{x-4} &\approx \frac{14}{+sm} = +\infty \end{aligned}\right\} \therefore \text{V.A. at } x=4$$

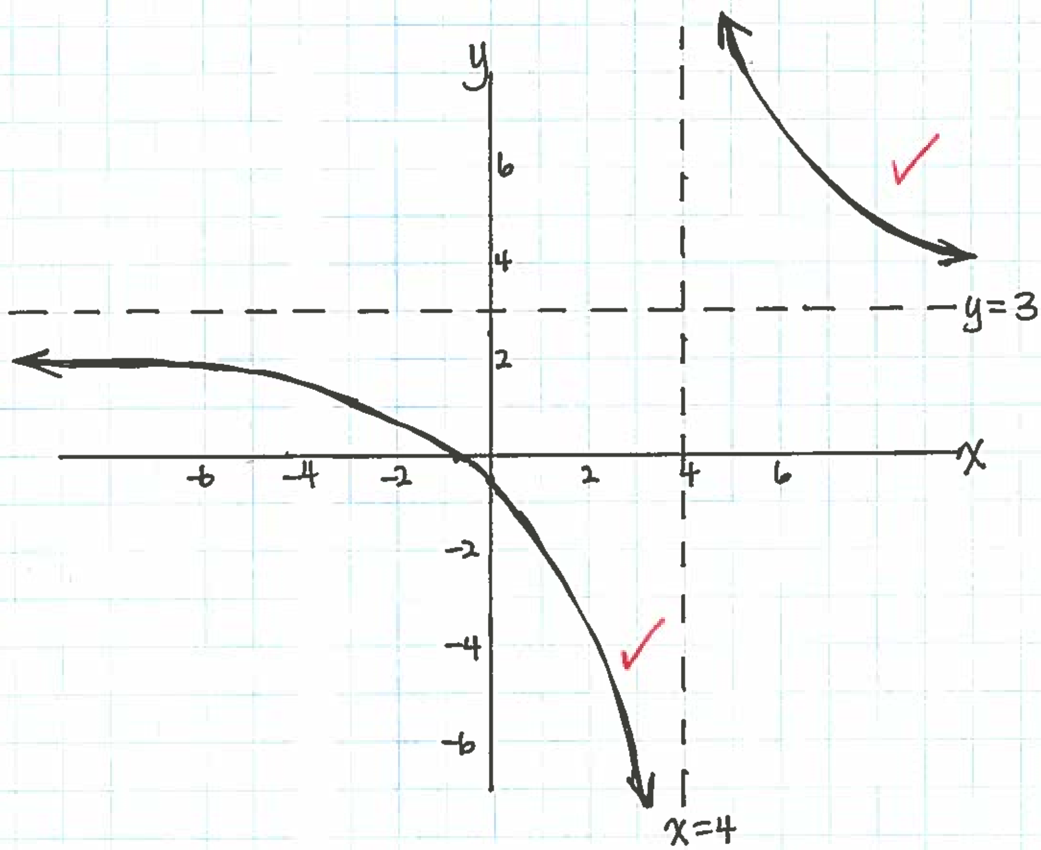
4. Asymptotes

$$\lim_{x\to\infty} \frac{3x+2}{x-4} \qquad \lim_{x\to-\infty} \frac{3x+2}{x-4}$$

$$= \lim_{x\to\infty} \frac{3+2/x}{1-4/x} \qquad = \lim_{x\to-\infty} \frac{3+2/x}{1-4/x}$$

$$= 3 \qquad\qquad = 3$$

$\therefore$ H.A. at $y=3$



4. a) $\log_5 50 - \log_5 0.4$

$= \log_5\left(\frac{50}{0.4}\right)$ ✓

5 $= \log_5 125$

$= 3$ ✓

b) $\log_9 27 + \log_4 \sqrt{8}$

$= \log_9(3^3) + \log_4 \sqrt{2^3}$

$= \log_9(3^2)^{3/2} + \log_4(2^2)^{3/4}$ ✓ ✓

$= \frac{3}{2} + \frac{3}{4}$

$= \frac{9}{4}$ ✓

5. $f(x) = \frac{x-1}{x+1}$ $\qquad g(x) = 2x+3$

$(f \circ g)(3) = f(g(3))$

$g(3) = 2(3)+3$
$= 9$ ✓

$= f(9)$ ✓

3 $= \frac{9-1}{9+1}$

$= \frac{8}{10} = \frac{4}{5}$ ✓

6. $f(x) = \frac{x^3-5x}{6x^7-4x^3}$

$f(-x) = \frac{(-x)^3-5(-x)}{6(-x)^7-4(-x)^3}$ ✓

$= \frac{-x^3+5x}{-6x^7+4x^3}$

$\because f(-x) = f(x)$ then $f(x)$ is even ✓

3 $= \frac{-(x^3-5x)}{-(6x^7-4x^3)}$

$= \frac{x^3-5x}{6x^7-4x^3}$ ✓ $= f(x)$

7. a) $f(x) = \log_5(x+3) - 2$

$y = \log_5(x+3) - 2$

Inverse: $x = \log_5(y+3) - 2$ ✓

$x+2 = \log_5(y+3)$

$5^{x+2} = y+3$

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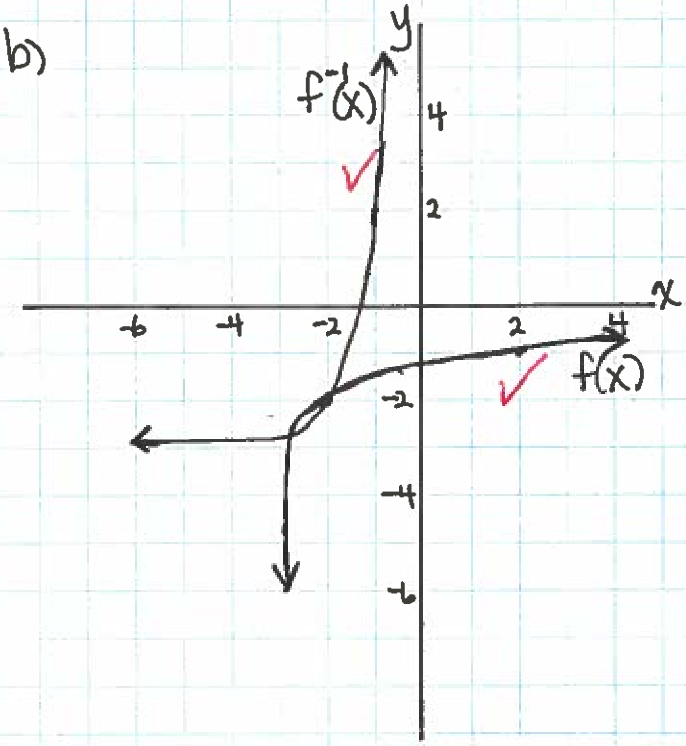
$y = 5^{x+2} - 3$ ✓

$\therefore f^{-1}(x) = 5^{x+2} - 3$

b)



8. a) Let $A = A_0 b^{t/d}$

$A = 10\,000$
$A_0 = 1250$
$b = 2$
$t = 5$ years

$10000 = 1250(2)^{5/d}$ ✓

$\frac{10000}{1250} = (2)^{5/d}$

$8 = (2)^{5/d}$ OR $\log_2 8 = \frac{5}{d}$

$2^3 = (2)^{5/d}$ ✓

3

$3 = \frac{5}{d}$

$3d = 5$

$d = \frac{5}{3}$ ✓

$\therefore$ The doubling period is $\frac{5}{3}$ years (ie. 20 months)

8. b) $P = 1250(2)^{t/5/3}$

1 $P = 1250(2)^{3t/5}$ ✓

c) $AROC = \frac{P(5) - P(0)}{5}$

$= \frac{10000 - 1250}{5}$ ✓

2 $= 1750$ ✓ people / year

d) When $P = 15000$, $15000 = 1250(2)^{3t/5}$ ✓

$12 = (2)^{3t/5}$

$\log_2 12 = \frac{3t}{5}$

3 $\frac{5}{3} \cdot \frac{\log 12}{\log 2} = t$ ✓

$t \doteq 5.97$ ✓

∴ It will take approx 6 years for the population to reach 15000.