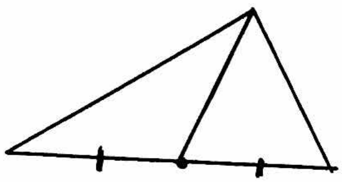
Note#________ Date:________________

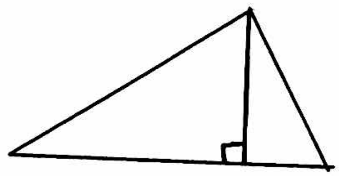
Medians, Altitudes, and Right Bisectors of a Triangle

median



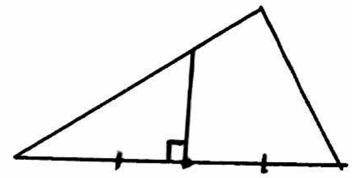
A median is a line segment that joins the midpt. of one side to the opposite vertex.

altitude



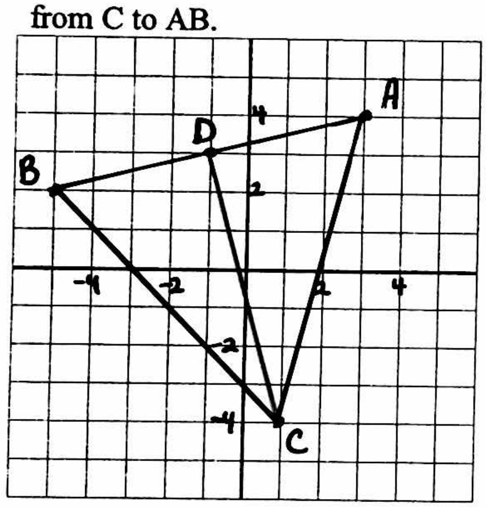
The altitude of a triangle is a line segment that joins a vertex to the opposite side at a right angle.

right bisector



The right bisector of a line segment is a line that passes through the midpoint and meets the side at a right angle.

Ex. 1. $\triangle ABC$ has vertices A(3, 4), B(-5, 2) and C(1, -4). Determine an equation for CD, the **median** from C to AB.



Find the midpt of AB

$$D = \text{midpt}_{AB} = \left(\frac{3+(-5)}{2}, \frac{4+2}{2}\right)$$

$$= \left(\frac{-2}{2}, \frac{6}{2}\right)$$

$$D = (-1, 3)$$

Find slope of CD

$C=(1,-4)$
$D=(-1,3)$

$$m_{CD} = \frac{y_2-y_1}{x_2-x_1}$$

$$= \frac{3-(-4)}{-1-1}$$

$$= \frac{7}{-2}$$

Sub into pt./slope form $m = \frac{y-y_1}{x-x_1}$

* use either pt. C or D.

Equation:

$$\frac{7}{-2} = \frac{y-3}{x-(-1)}$$

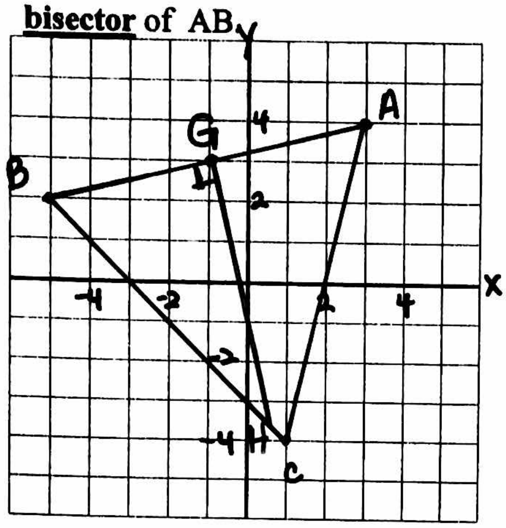
$$7x+7 = -2y+6$$

$$\boxed{7x+2y+1=0}$$

*Note There are 2 other medians in the triangle: one from B to AC and one from A to BC.

right bisector = perpendicular bisector

Ex. 2. $\triangle ABC$ has vertices A(3, 4), B(-5, 2) and C(1, -4). Determine an equation for $\boxed{GH}$, the **right bisector** of AB.



From the last example, the midpt of AB is (-1, 3). = G

Find slope of AB:

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{2-4}{-5-3}$$

$$= \frac{-2}{-8} = \frac{1}{4}$$

Since $GH \perp AB$ then $m_{GH} = -4$

negative reciprocal

Equation: $m_{GH} = -4$

$G = (-1, 3)$

$$m = \frac{y - y_1}{x - x_1}$$

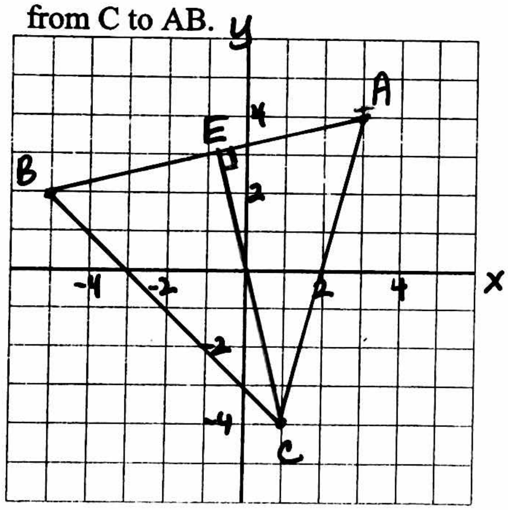
$$\frac{4}{-1} = \frac{y-3}{x+1}$$

$$4x + 4 = -y + 3$$

$$\boxed{4x + y + 1 = 0}$$

* there are 3 possible right bisectors in any given triangle.

Ex. 3. $\triangle ABC$ has vertices A(3, 4), B(-5, 2) and C(1, -4). Determine an equation for CE, the **altitude** from C to AB.



From the example above, the slope of AB is $\frac{1}{4}$.

Since CE is $\perp$ to AB then $m_{CE} = -4$.

Equation: through C(1, -4)

$$m = \frac{y - y_1}{x - x_1}$$

$$\frac{4}{-1} = \frac{y+4}{x-1}$$

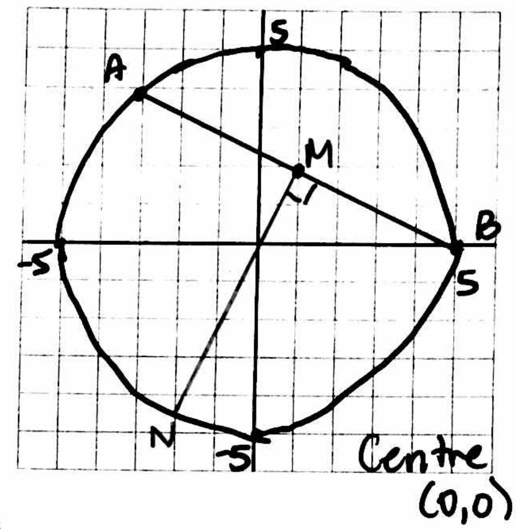
$$4x - 4 = -y - 4$$

$$\boxed{4x + y = 0}$$

* There are 3 possible altitudes in any given triangle

Ex. 4 The points **A**(-3, 4) and **B**(5, 0) are endpoints of a ***chord of a circle*** with centre (0, 0) and radius 5. Verify that the centre of the circle lies on the right bisector of this chord.

Find the equation of right bisector to AB:



$$Midpt_{AB} = \left(\frac{-3+5}{2}, \frac{4+0}{2}\right)$$

$$= \left(\frac{2}{2}, \frac{4}{2}\right)$$

$$M = (1, 2)$$

Find slope of AB:

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{0-4}{5+3}$$

$$= \frac{-4}{8} = \frac{-1}{2}$$

Since $MN \perp AB$ then $m_{MN} = 2$

Equation:

$m = 2$

$M = (1, 2)$

$$m = \frac{y - y_1}{x - x_1}$$

$$\frac{2}{1} = \frac{y-2}{x-1}$$

$$2x - 2 = y - 2$$

$$\boxed{2x - y = 0}$$

Verify pt. (0,0) is on the line $2x - y = 0$

$$LS = 2x - y$$
$$= 2(0) - (0)$$
$$= 0$$

$$RS = 0$$

∴ Since LS = RS the pt (0,0) is on the right bisector $2x - y = 0$