# Financial Applications – Simple & Compound Interest

## Simple Interest

Suppose you just won **$1 000**. You decide to put the money in the bank to earn interest. The type of account you chose pays **simple interest**. With simple interest, the amount of interest earned is determined as a percentage of the original principal.

**Simple Interest** is calculated using the following formula:

***I = Prt***, where:

*I* is the interest (in dollars)
*P* is the principal (amount of money you put into the account)
*r* is the annual rate of interest (as a decimal)
*t* is the time that the money is in the account (measured in years)

The **future amount (A)** of the investment is given by: ***A = P + I***

**Eg. 1** Consider $1 000 in the account that pays 2%/**a** (per annum or per year). You leave the money in the account for one year. How much interest do you earn, and what is the balance in the account?

list variables
$P = \$1000$
$r = .02$
$t = 1$

$I = Prt$
$= 1000(.02)(1)$
$= 20$

∴ You earn $20 interest

Now, calculate the total amount in the account after one year:

$A = P + I$
$= 1000 + 20$
$= 1020$

∴ The balance after 1 year is $1020.

Let's consider what the balance will look like if the money is left in the account for 5 years:

| Year | Interest Earned (I=Prt) (for 1 year) | Balance in Account (A=P+I) |
|---|---|---|
| 0 | | $1000 |
| 1 | 1000(.02) = 20 | 1000+20 = $1020 |
| 2 | 1000(.02) = 20 | 1020+20 = 1040 |
| 3 | 1000(.02) = 20 | 1040+20 = 1060 |
| 4 | 1000(.02) = 20 | 1060+20 = 1080 |
| 5 | 1000(.02) = 20 | 1080+20 = $1100 |

Note that the terms in the "Balance" column form an **arithmetic sequence** with a **common difference of $20**. This corresponds to the amount of interest earned in any **one year**. If we consider a sequence **starting with the balance after Year 1** (not the inital amount deposited), then the balance after 5 years could be calculated as follows:

$d = 20$
$a = 1020$
$n = 5$

$t_n = a + (n-1)d$
$t_5 = 1020 + (5-1)(20)$
$= 1020 + (4)(20)$
$= \$1100$

$t = \frac{5}{12}$ (since time must be expressed as a fraction of a year)

**Eg. 2** You borrowed $4000 from the bank at a rate of 6.5%/a (per year) for 5 months. How much must you repay in interest and in total?

$P = \$4000$

$r = .065$

$t = \frac{5}{12}$

$I = Prt$

$= 4000(.065)\left(\frac{5}{12}\right)$

$= \$108.33$

$A = P + I$

$= 4000 + 108.33$

$= 4108.33$

∴ You earn $108.33 interest and you would have a balance of $4108.33

**Compound Interest**

Suppose you decide to switch your $1000 investment to a **compound interest** account.

In a **simple interest** account, you earn the same amount of interest every time period, since the interest is always calculated on the original principal. In a **compound interest** account, the interest earned is added to the original principal every time period. In the next time period, the interest is calculated on the principal plus the previous interest earned. So basically you can earn interest on the principal and the previous interest.

Determine the amount of an investment of $1000 at 2%/a compounded annually for five years:

| Year | Interest Earned During the Year | Balance at the end of the Year |
|---|---|---|
| 0 | | $1000 |
| 1 | 1000(.02)=20 | 1000+20= 1020 |
| 2 | 1020(.02)= 20.40 | 1020+20.40= 1040.40 |
| 3 | 1040.40(.02)=20.81 | 1040.40+20.81=1061.21 |
| 4 | 1061.21(.02)=21.22 | 1061.21+21.22=1082.43 |
| 5 | 1082.43(.02)=21.65 | 1082.43+21.65=1104.08 |

Notice that each year the interest is calculated by taking the interest rate (2%) times the balance from the previous year.

**Comparing simple interest and compound interest:**

From the previous examples, the amount of a $1000 investment at 2%/a after 5 years is:

Simple Interest Account = $1100

Compound Interest Account = $1104.08

Notice that the interest earned in the compound interest account is 4.08 more than the simple interest account, simply because interest is calculated on the previous principal and interest together. Keep in mind that if we continued the calculations for another 5 years, the difference would be even greater.

Now let's take a closer look at the method used to derive compound interest;

| **Principal** | **Year 1** | **Year 2** | **Year 3** | **...** | **Year n** |
|---|---|---|---|---|---|
| P | P(1+i) | P(1+i)(1+i) | P(1+i)(1+i)(1+i) | ... | $P(1+i)^n$ |

This generates a **geometric sequence** with **a** = P(1+i) and ***r*** = (1 + i).

This would give us $t_n = P(1+i)^n$

or $\boxed{A = P(1+i)^n}$

For **compound interest,** the **Amount** is calculated using the formula $A=P(1+i)^n$, where:

***i*** is the interest rate per compounding period (as a decimal) $i = \frac{\text{annual interest rate}}{\#\text{compounding periods}}$
***P*** is the principal amount of money invested or borrowed
***A*** is the amount you end up with or owe
***n*** is the the number of compounding periods $n = \text{time (years)} \times \#\text{comp. periods}$

****Note**: A **compounding period** refers to how often the interest is calculated and added to the account.

If the interest is compounded **annually**, then it is compounded **once per year**.
If the interest is compounded **semi-annually**, then it is compounded **twice per year**.
If the interest is compounded **quarterly**, then it is compounded **4 times per year**.
If the interest is compounded **monthly**, then it is compounded **12 times per year**

To calculate the interest earned on the account: **I = A – P**

**Eg. 1** Calculate the interest rate per compounding period (**i**) and the total number of compounding periods (**n**) for each of the following:

**a)** 8%/a, compounded quarterly for 3 years

$i = \frac{.08}{4} = .02$

compounded quarterly = 4 times/year

$n = 3 \times 4 = 12$

**b)** 6%/a, compounded monthly for 2.5 years

$i = \frac{.06}{12} = .005$

$n = 2.5 \times 12$

$n = 30$

**c)** 7% compounded semi-annually for 18 months

$i = \frac{.07}{2} = .035$

$n = 1.5 \times 2 = 3$

18 mon = 1.5 years

**Eg. 2** Suppose you invest $5000 in an account that pays 3%/**a** (per annum or per year) **compounded semi-annually**. How much do you have in your account after 3 years? How much interest did you earn?

$P = \$5000$

$i = \frac{.03}{2} = .015$

$n = 3 \times 2 = 6$

$A = P(1+i)^n$
$= 5000(1+.015)^6$
$= 5467.22$

$I = A - P$
$= \$467.22$

∴ You have $5467.22 in your account which includes $467.22 interest

**Eg. 3** Calculate the amount after 2 years if $1000 is invested in an account earning 6%/a, if the interest is:

**a)** compounded annually

$i = .06 \quad n = 2$

$A = 1000(1+.06)^2$
$= \$1123.60$

**b)** compounded quarterly

$i = \frac{.06}{4} = .015$

$n = 2 \times 4 = 8$

$A = 1000(1+.015)^8$
$= 1126.49$

**c)** compounded monthly

$i = \frac{.06}{12} = .005$

$n = 2 \times 12 = 24$

$A = 1000(1+.005)^{24}$
$= 1127.16$

Notice that the amount of money in the account, and the amount of interest earned increases when the interest is compounded more frequently.