Sequences

A sequence is a set of numbers, separated by commas with a definite order.

You are familiar with many sequences, such as:

1, 2, 3, 4, 5, ... ($\mathbb{N}$ - natural numbers)

1, 4, 9, 16, 25, ... (sequence of perfect squares)

A sequence that has a last term is called a finite sequence. A sequence with no last term, but continues on indefinitely is called an infinite sequence. We use three dots (...) to indicate the sequence goes forever.

Each number in the sequence is called a term. We use the following symbols to represent terms:

$t_1, t_2, t_3 .. t_n$ t_1 = first term t_2 = second term t_n = n^{th} term

Using the sequence of perfect squares, we could write: $t_1 = 1,\ t_2 = 4,\ t_3 = 9,\ t_4 = 16, \ldots$

However, it also could be written as: $t_1 = 1^2,\ t_2 = 2^2,\ t_3 = 3^2,\ t_4 = 4^2, \ldots.$ $t_n = n^2$

Notice how the subscripts of the t's are actually used in the calculation of each term. We do this so that we can generate a formula to find any term in the sequence. We call this formula the general term.

For our example, the general term is $t_n = n^2$. (Trial and error is used to see the pattern)

Ex.1 Given the general term $t_n = 4n - 2$, find the first three terms and the 12th term.

$t_1 = 4(1) - 2$
$= 2$

$t_2 = 4(2) - 2$
$= 6$

$t_3 = 4(3) - 2$
$= 10$

$t_{12} = 4(12) - 2$
$= 46$

Ex. 2 Find the general term of the following sequences

term # → 1 2 3 4

a) 4, 7, 10, 13, ...

+3 +3 +3

Try to relate each term to its corresponding term #. Here we notice that each term is 1 more than a multiple of 3.

$\therefore t_n = 3n + 1.$

b) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$

$t_n = \frac{n}{n+1}$