Recursion Formulas

Previously, we defined a formula for the general term (n^{th} term) of an arithmetic or geometric sequence. For an arithmetic sequence, the n^{th} term is defined using $\mathbf{t_n = a + (n - 1)d}$. For a geometric term, the n^{th} term is defined using $\mathbf{t_n = ar^{n-1}}$. These formulas are known as **explicit formulas** because they can be used to calculate any term in a sequence without knowing the previous term.

Sometimes it is more convenient to calculate a term in a sequence from one or more previous terms. Formulas that do this are called **recursion formulas**. A recursion formula consists of at least two parts: the first part of the formula defines the first term(s) of the sequence, while the second part defines how to calculate subsequent terms from the terms before it.

A "famous" sequence defined using recursion formulas is the **Fibonacci Sequence**. The terms of this sequence are: 1, 1, 2, 3, 5, 8, 13, ... Notice that after the first two terms, subsequent terms are equal to the sum of the two terms before it. Using a recursion formula, we would define this sequence as:

$t_1 = 1, \quad t_2 = 1 \quad$ and $\quad t_n = t_{n-1} + t_{n-2}$

This means that the first two terms in the sequence are 1 and each successive term in the sequence is generated by adding the two previous terms together.

Note: In a **recursion formula** you must always specify the first term or terms as required by the formula. The formula always expresses the general term $\mathbf{t_n}$ in terms of the previous term or terms.

Eg. 1 Write the first four terms of the sequence defined by the following recursion formula:

$$t_1 = 2 \,;\; t_n = t_{n-1} + 3$$

Solution: The first term is given in the formula: $t_1 = 2$.

The second term is calculated from the first using $t_n = t_{n-1} + 3$

$$\begin{aligned} t_2 &= t_{2-1} + 3 \\ &= t_1 + 3 \\ &= 2 + 3 \qquad \text{(Sub } t_1 = 2\text{)} \\ &= 5 \end{aligned}$$

The third and fourth terms are calculated similarly as follows:

$$\begin{aligned} t_3 &= t_2 + 3 \\ &= 5 + 3 \\ &= 8 \end{aligned} \qquad\qquad \begin{aligned} t_4 &= t_3 + 3 \\ &= 8 + 3 \\ &= 11 \end{aligned}$$

Therefore, the first four terms are 2, 5, 8, 11.

Eg. 2 Define an explicit formula and a recursion formula for the geometric sequence 2, 6, 18, 54, ...

Solution: For the **explicit formula**, use $t_n = ar^{n-1}$ with $a = 2$, $r = 3$

Therefore, the explicit formula is $t_n = 2(3)^{n-1}$

For the **recursion formula**, first define the first term: $t_1 = 2$

Since subsequent terms are calculated by multiplying the previous term by 3, then $t_n = 3t_{n-1}$

Therefore, the recursion formula is $t_1 = 2 \,;\, t_n = 3t_{n-1}$

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