Max/Min Word Problems – Additional Review

1. Determine the absolute maximum and minimum points for the function:

    a) $f(x) = \dfrac{1+x}{1-x}$ for $2 \le x \le 5$

    b) $g(x) = 2\sin x + \cos 2x$ for $-\dfrac{\pi}{2} \le x \le \dfrac{\pi}{2}$

2. A chain of stores has been selling a line of music players for $50 each and has been averaging sales of 8000 players per month. They decide to increase the price, but their market research indicates that for each $1 increase in price, sales will fall by 100. Determine the price that will maximize their revenue.

3. A company determines that the cost, in dollars, of producing x items is
    $C(x) = 280000 + 12.5x + 0.07x^2$.
    a) Determine the average cost and the marginal cost of producing 1000 items.
    b) At what production level will the average cost be a minimum?
    c) What is the minimum average cost?

4. The lifeguard at a public beach has 400 m of rope available to lay out a rectangular swimming area using the straight shoreline as one side of the rectangle.
    a) Determine the dimensions of the rectangle that will maximize the swimming area.
    b) To ensure the safety of swimmers, the lifeguard decides that nobody should be more than 50 m from shore. What should the dimensions of the swimming area be with this added restriction?

5. A can is to be made to hold a litre of oil. Determine the radius of the can that will minimize the cost of the metal to make the can. (Hint: 1L = 1000 cm^3).

6. A Norman window has the shape of a rectangle capped by a semicircle, with the diameter of the semicircle equal to the width of the rectangle. If the perimeter of the window is 8 m, determine the width of the window that will admit the greatest amount of light.

7. Find the area of the largest rectangle, with sides parallel to the axes, that can be inscribed in the ellipse $x^2 + 4y^2 = 4$ as shown in the diagram below.
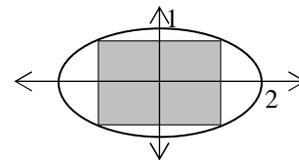
8. A boat leaves a dock at noon and heads west at a speed of 25 km/h. Another boat heads north at 20 km/h and reaches the same dock at 1:00 pm. At what time were the boats closest to each other?

9. A cable TV company is laying cable in an area with underground utilities. Two subdivisions are located on opposite sides of Willow Creek, which is 100 m wide. The company has to connect points P and Q with cable, where Q is on the north bank, 1200 m east of P. It costs $40/m to lay cable underground and $80/m to lay cable underwater. Where should they lay the cable to minimize the total cost?
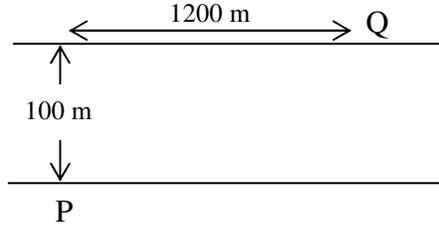
10. A rectangle has dimensions 3 cm by 5 cm. The rectangle is enclosed by another rectangle ABCD as shown in the diagram below.

a) Determine the measure of angle θ so that rectangle ABCD has maximum area.

b) What are the dimensions of ABCD with maximum area?
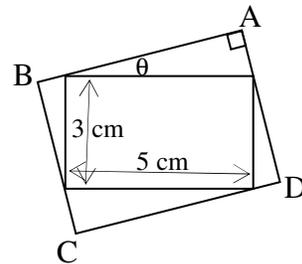
Answers:

1. a) max: $\left(5,-\frac{3}{2}\right)$; min: $(2,-3)$ b) max: $\left(\frac{\pi}{6},\frac{3}{2}\right)$; min: $\left(-\frac{\pi}{2},-3\right)$ 2. $65

3. a) $362.50/item; $152.50/item   b) 2000 items   c) $292.50/item
4. a) 100 m x 200 m   b) 50 m x 300 m   5. 5.4 cm   6. 2.24 m
7. 4 units^2 8. approx 12:23 pm
9. underground to a pt. 1142 m east of P, then underwater to pt. Q

10. a) $\theta = \dfrac{\pi}{4}$ rad.   b) $4\sqrt{2}$ cm $\times$ $4\sqrt{2}$ cm