MAX – min PROBLEMS

1. The sum of two natural numbers is 12. Find the numbers if their product is a *maximum*.

2. Two numbers have a difference of 8. Find the numbers if their product is a *minimum*.

3. The sum of two numbers is 16. Find the numbers if the sum of their squares is a *minimum*.

4. Two numbers have a difference of 16. Find the numbers if the result of adding their sum and their product is a *minimum*.

5. Find the number which exceeds its square by the *greatest* possible amount.

6. A flare is projected upwards with a velocity of 60m/s. The function that relates the height of the flare in metres to the time in seconds is:
$$h(t) = -5t^2 + 60t + 2$$
Determine the *maximum* height the flare reaches, and the time at which it reaches that height.

7. The height of a Tiger Woods drive can be obtained by the function:
$$h(t) = -\frac{1}{4}t^2 + 2t + 26$$
where h is the height in metres and t is the time in seconds. Determine the *maximum* height of Tiger's drive, and the time at which it reaches that height.

8. A lifeguard marks off a rectangular swimming area at a beach with 200m of rope. What is the *greatest* area she can enclose?

9. A rectangular area is enclosed by a fence and divided by another section of fence which is parallel to two of its sides. If the 600m of fence used encloses a *maximum* area, what are the dimensions of the enclosure?

10. What is the *maximum* area of a triangle whose sum of its base and height is 15cm.

11. A theatre which seats 2000 people charges $10/ticket and always sells out. A survey indicates that if the ticket price is increased, the number sold will decrease by 100 for every dollar of increase. What ticket price would result in the *greatest* revenue?

12. A bus company carries about 20,000 riders per day at a fare of $0.90. A survey indicates that if the fare is decreased, the number of riders will increase by 2000 for every $0.05 of decrease. What fare would result in the *greatest* revenue?

Max / Min Word Problems

1. Let x be one number. Let y be the other number.

Given: $x+y=12$
$y=12-x$

max $P=x\cdot y$
$=x(12-x)$
$=12x-x^2$

Complete the square: $P=-x^2+12x$
$=-(x^2-12x+36-36)$
$=-(x-6)^2+36$

$x=6$:
$y=12-6=6$

$\therefore$ The maximum product is 36 when the numbers are 6 and 6.

② Let x be one number and let y be the other number.

Given: $x-y=8$
$x=8+y$

min $P=x\cdot y$
$=(8+y)y$
$=8y+y^2$

Complete the square: $P=y^2+8y+16-16$
$=(y+4)^2-16$

$y=-4$: $x=8-4$
$=4$

$\therefore$ The minimum product is -16 when the numbers are 4 and -4.

③ Let x be one number and let y be the other number.

Given: $x+y=16$
$y=16-x$

min. $S=x^2+y^2$
$=x^2+(16-x)^2$
$=x^2+256-32x+x^2$
$=2x^2-32x+256$

Complete the square:
$S=2(x^2-16x)+256$
$=2(x-16x+64-64)+256$
$=2(x-8)^2-128+256$
$=2(x-8)^2+128$

$\therefore$ The minimum sum is 128 when the numbers are 8 and 8.

$x=8$
$y=16-8=8$

④ Let x be one number and let y be the other number.

Given: $x-y=16$

$x=16+y$

min $S=(x+y)+xy$

$=(16+y+y)+(16+y)y$

$=16+2y+16y+y^2$

$=y^2+18y+16$

Complete the square:

$S=y^2+18y+81-81+16$

$=(y+9)^2-65$

$y=-9$

$x=16-9=7$

∴ The numbers are -9 and 7.

⑤ Let x be the number

max $D=x-x^2$

$=-(x^2-x)$

Complete Square:

$=-(x^2-x+\frac{1}{4}-\frac{1}{4})$

$=-(x-\frac{1}{2})^2+\frac{1}{4}$

∴ The number $\frac{1}{2}$ exceeds its square by the greatest amount.

⑥ $h=-5t^2+60t+2$

$=-5(t^2-12t)+2$

$=-5(t^2-12t+36-36)+2$

$=-5(t-6)^2+180+2$

$=-5(t-6)^2+182$

∴ The maximum height reached is 182m when $t=6$ seconds.

⑦ $h=-\frac{1}{4}t^2+2t+26$

$h=-\frac{1}{4}(t^2-8t)+26$

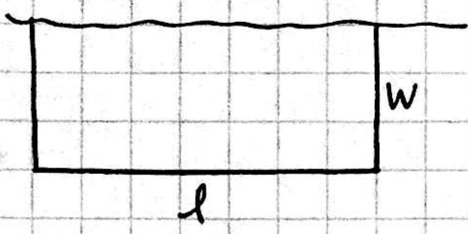
$=-\frac{1}{4}(t^2-8t+16-16)+26$

$=-\frac{1}{4}(t-4)^2+4+26$

$=-\frac{1}{4}(t-4)^2+30$

∴ The maximum height is 30m when $t=4$ seconds.

⑧



Let l be the length and w be the width.

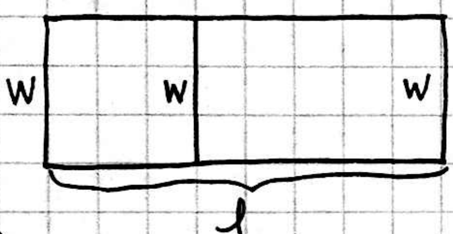
$$\max A = l \cdot w$$
$$= (200-2w)\cdot w$$
$$= 200w - 2w^2$$
$$= -2w^2 + 200w$$

Given: $2w + l = 200$
$l = 200 - 2w$

$$A = -2(w^2 - 100w + 2500 - 2500)$$
$$= -2(w-50)^2 + 5000$$

∴ The maximum area that can be enclosed is 5000 m^2 when the width is 50 m and the length is 100 m.

⑨



Let l be the length of the area.
Let w be the width

$$\max A = l \cdot w$$
$$= \left(300 - \frac{3}{2}w\right)w$$
$$= 300w - \frac{3}{2}w^2$$
$$= -\frac{3}{2}w^2 + 300w$$

Given: $3w + 2l = 600$
$2l = 600 - 3w$
$l = 300 - \frac{3}{2}w$

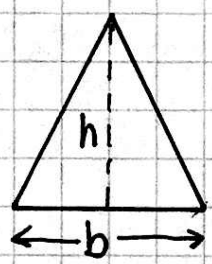
Complete the square:

$$A = -\frac{3}{2}(w^2 - 200w + 10000 - 10000)$$
$$= -\frac{3}{2}(w-100)^2 + 15000$$

∴ The max area is 15000 m^2 when the width is 100 m and the length is 150 m.

Find l: $l = 300 - \frac{3}{2}(100)$
$= 300 - 150$
$= 150$

⑩ Let b be the length of the base.
Let h be the height



Given $b+h=15$
$h=15-b$

max $A=\frac{1}{2}b\cdot h$
$=\frac{1}{2}b(15-b)$
$=\frac{15}{2}b-\frac{1}{2}b^2$

Complete square: $A=-\frac{1}{2}(b^2-15b)$
$=-\frac{1}{2}\left(b^2-15b+\frac{225}{4}-\frac{225}{4}\right)$
$=-\frac{1}{2}\left(b-\frac{15}{2}\right)^2+\frac{225}{8}$

$\therefore$ The maximum area is $\frac{225}{8}=28.125\ cm^2$ when the base is $\frac{15}{2}=7.5cm$ and the height is $7.5cm$

⑪ Let x be the increase in ticket price.

Revenue = # tickets sold × ticket price

$R=(2000-100x)(10+x)$
$=20000+2000x-1000x-100x^2$
$=-100x^2+1000x+20000$

Complete the square:

$R=-100(x^2-10x)+20000$
$=-100(x^2-10x+25-25)+20000$
$=-100(x-5)^2+2500+20000$
$=-100(x-5)^2+22500$

$\therefore$ The maximum revenue is \$22500 when the price <u>increase</u> is \$5 (ie. ticket price should be \$15)

12. Let x rep. the number of \$0.05 decreases.

$$
\begin{aligned}
\text{Rev} &= \#\text{sold} \times \text{price} \\
&= (20{,}000 + 2000x)(0.90 - 0.05x) \\
&= 18000 - 1000x + 1800x - 100x^2 \\
&= -100x^2 + 800x + 18000 \\
&= -100(x^2 - 8x) + 18000 \\
&= -100(x^2 - 8x + 16 - 16) + 18000 \\
&= -100(x^2 - 8x + 16) + 1600 + 18000 \\
&= -100(x-4)^2 + 19600
\end{aligned}
$$

$$\therefore x = 4$$

$$
\begin{aligned}
\therefore \text{price} &= 0.90 - 0.05(4) \\
&= 0.90 - 0.2 \\
&= 0.70
\end{aligned}
$$

$\therefore$ the fare should be \$0.70 to maximize revenue