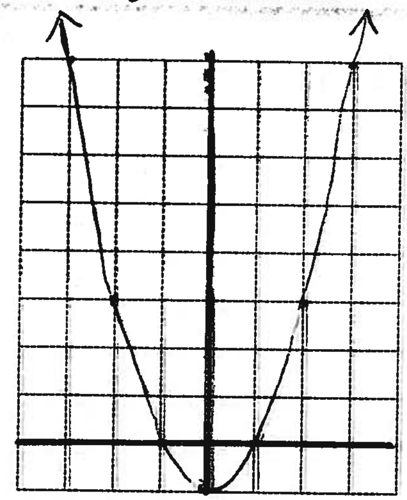
The Limit of a Function

Graph the function: $f(x) = x^2 - 1$



Complete the following table to examine the values of $y=f(x)$ as the x values get close to 2.

$x \to 2$ $\qquad$ $x \to 2$

x	1	1.5	1.9	1.99	1.999	2	2.001	2.01	2.1	2.5	3
y	0	1.25	2.61	2.9601	2.996	3	3.004	3.0401	3.41	5.25	8

$f(x) \to 3$ $\qquad$ $f(x) \to 3$

Notice that:

As the x values approach 2 from the left, the values of $f(x)$ approach 3

As the x values approach 2 from the right, the values of $f(x)$ approach 3

Notation:

$$\lim_{x \to 2^-} f(x) = 3$$

from left

Notation:

$$\lim_{x \to 2^+} f(x) = 3$$

from right

When the limit from the left and the limit from the right are equal, we write:

$$\lim_{x \to 2} x^2 - 1 = \lim_{x \to 2} f(x) = 3$$

(2 sided limit)

In general:

The limit $\lim_{x \to a} f(x) = L$ if and only if $\lim_{x \to a^-} f(x) = L = \lim_{x \to a^+} f(x)$.

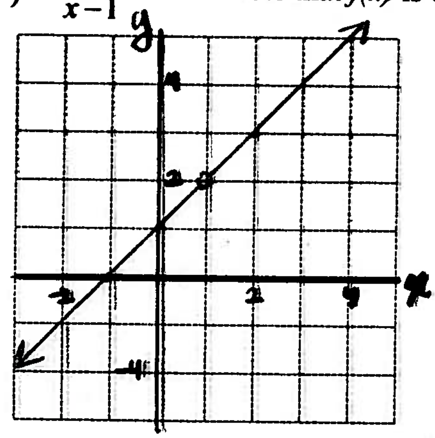
Otherwise $\lim_{x \to a} f(x)$ does not exist.

Notice, in the above function, $f(2) = 3$ $= \lim_{x \to 2} f(x)$ but this is not always the case.

Eg. 2 Graph the function: $f(x)=\frac{x^2-1}{x-1}$ *Note that $f(x)$ is undefined when $x=1$

$f(x)=\frac{(x-1)(x+1)}{x-1}$

$=x+1$



*note hole at (1,2)

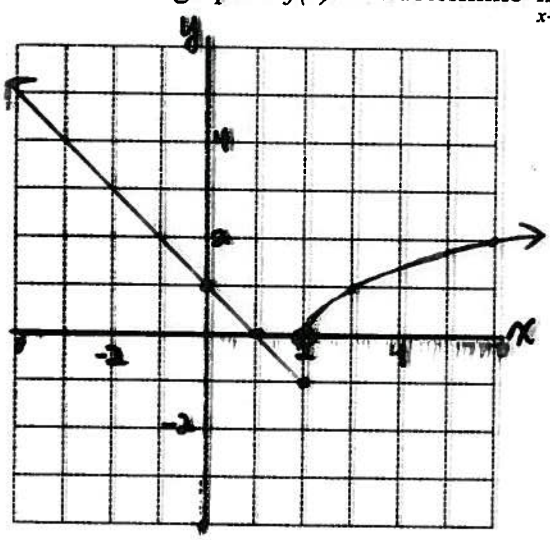
From your graph, determine:

a) $\lim_{x\to 1^-}\frac{x^2-1}{x-1}=2$ b) $\lim_{x\to 1^+}\frac{x^2-1}{x-1}=2$ c) $\lim_{x\to 1}\frac{x^2-1}{x-1}=2$

even though $f(1)$ is undefined.

Eg. 3 Given: $f(x)=\begin{cases}\sqrt{x-2} & \text{if } x>2\\ -x+1 & \text{if } x\le 2\end{cases}$ $y=-x+1$

Sketch the graph of $f(x)$ and determine $\lim_{x\to 2} f(x)$ if it exists.



$\lim_{x\to 2^-} f(x) = -1$

$\lim_{x\to 2^+} f(x) = 0$

Since $\lim_{x\to 2^-} f(x) \neq \lim_{x\to 2^+} f(x)$

then $\lim_{x\to 2} f(x)$ does not exist.

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