Limit Questions Worksheet - Solutions

(1) $\lim_{x\to 1} \frac{x+1}{x^2+1}$

$= \frac{(1)+1}{(1)^2+1}$

$= \frac{2}{2} = 1$

(2) $\lim_{x\to -3} \frac{x^2-9}{x+3}$

$= \lim_{x\to -3} \frac{(x+3)(x-3)}{(x+3)}$

$= \lim_{x\to -3} x-3$

$= (-3)-3 = -6$

(3) $\lim_{x\to 0} \frac{3x-4}{x^2-12}$

$= \frac{3(0)-4}{(0)^2-12}$

$= \frac{1}{3}$

(4) $\lim_{x\to 2^+} \frac{x+2}{x^2-4}$

$\doteq \frac{4}{(+sm)} = +\infty$

(5) $\lim_{x\to \infty} \frac{2x^2-5x+1}{3x^2-2x-3}$

$= \lim_{x\to \infty} \frac{2 - 5/x + 1/x^2}{3 - 2/x - 3/x^2}$

$= \frac{2}{3}$

(6) $\lim_{x\to 1} \frac{x^3-1}{x^3-x^2-4x+4}$

$= \lim_{x\to 1} \frac{(x-1)(x^2+x+1)}{x^2(x-1)-4(x-1)}$

$= \lim_{x\to 1} \frac{(x-1)(x^2+x+1)}{(x-1)(x^2-4)}$

$= \frac{(1)^2+(1)+1}{(1)^2-4}$

$= \frac{3}{-3} = -1$

(7) $\lim_{x\to \infty} \left(\frac{4}{5}\right)^x$

$= 0$

(8) $\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2}$

$= \lim_{x\to 4} \frac{(x-4)(\sqrt{x}+2)}{x-4}$

$= \lim_{x\to 4} \sqrt{x}+2$

$= \sqrt{4}+2$

$= 4$

(9) $\lim_{x\to 0} \frac{3^x}{2^x}$

$= \frac{3^0}{2^0} = 1$

(10) $\lim_{h\to 0} \frac{\sqrt{a+h}-\sqrt{a}}{h} \cdot \frac{\sqrt{a+h}+\sqrt{a}}{\sqrt{a+h}+\sqrt{a}}$

$= \lim_{h\to 0} \frac{(a+h)-a}{h(\sqrt{a+h}+\sqrt{a})}$

$= \lim_{h\to 0} \frac{h}{h(\sqrt{a+h}+\sqrt{a})}$

$= \lim_{h\to 0} \frac{1}{\sqrt{a+h}+\sqrt{a}}$

$= \frac{1}{\sqrt{a}+\sqrt{a}}$

$= \frac{1}{2\sqrt{a}} \cdot \frac{\sqrt{a}}{\sqrt{a}} = \frac{\sqrt{a}}{2a}$

(11) $\lim_{x\to\infty} \frac{(0.5)^x}{1+(0.3)^x}$

$= \frac{0}{1+0} = 0$

(12) $\lim_{x\to -1^-} \frac{1}{(x+1)^3}$

$\doteq \frac{1}{(-5m)^3} = -\infty$

(13) $\lim_{x\to\infty} \frac{x^2+2x}{x}$

$= \lim_{x\to\infty} x+2 = \infty$

(14) $\lim_{x\to 0} \frac{x^2+2x}{x}$

$= \lim_{x\to 0} \frac{x(x+2)}{x}$

$= 0+2 = 2$

(15) $\lim_{h\to 0} \frac{(2+h)^2-4}{h}$

$= \lim_{h\to 0} \frac{[(2+h)-2][(2+h)+2]}{h}$

$= \lim_{h\to 0} \frac{h[(2+h)+2]}{h}$

$= 4$

(16) $\lim_{x\to\infty} 100^{1/x}$

$= 100^0 = 1$

(17) $\lim_{x\to 3} \frac{\frac{1}{x}-\frac{1}{3}}{x-3}$

$= \lim_{x\to 3} \frac{3-x}{3x} \cdot \frac{1}{x-3}$

$= \lim_{x\to 3} \frac{-1(x-3)}{3x} \cdot \frac{1}{x-3} = \frac{-1}{9}$

(18) $\lim_{x\to\infty} \frac{4x-5}{7x^2-2x+4}$

$= \lim_{x\to\infty} \frac{4/x - 5/x^2}{7 - 2/x + 4/x^2}$

$= \frac{0}{7} = 0$

(19) $\lim_{x \to -2^-} \frac{x^2-x-2}{x^2+3x+2}$

$= \lim_{x \to -2^-} \frac{(x-2)(x+1)}{(x+2)(x+1)}$

$= \frac{-4}{(-5m)} = +\infty$

(20) $\lim_{x \to \infty} x^2 - x^3$

$= \lim_{x \to \infty} x^3(1-x)$

$(+\infty)(-\infty)$

$= -\infty$

(21) $\lim_{x \to 1} \frac{1-x^3}{2-\sqrt{x^2+3}}$

$= \lim_{x \to 1} \frac{(1-x)(1+x+x^2)}{2-\sqrt{x^2+3}} \cdot \frac{2+\sqrt{x^2+3}}{2+\sqrt{x^2+3}}$

$= \lim_{x \to 1} \frac{(1-x)(1+x+x^2)(2+\sqrt{x^2+3})}{4-(x^2+3)}$

$= \lim_{x \to 1} \frac{(1-x)(1+x+x^2)(2+\sqrt{x^2+3})}{1-x^2}$

$= \lim_{x \to 1} \frac{(1-x)(1+x+x^2)(2+\sqrt{x^2+3})}{(1-x)(1+x)}$

$= \frac{(1+(1)+(1)^2)(2+\sqrt{1^2+3})}{(1+1)}$

$= \frac{3(4)}{2} = 6$

(22) $\lim_{x\to 4} \dfrac{\frac{1}{\sqrt{x}} - \frac{1}{2}}{x-4}$

$= \lim_{x\to 4} \dfrac{2-\sqrt{x}}{2\sqrt{x}} \cdot \dfrac{1}{x-4} \cdot \dfrac{2+\sqrt{x}}{2+\sqrt{x}}$

$= \lim_{x\to 4} \dfrac{4-x^{-1}}{(2\sqrt{x})(x-4)(2+\sqrt{x})}$

$= \lim_{x\to 4} \dfrac{-1}{(2\sqrt{x})(2+\sqrt{x})}$

$= \dfrac{-1}{(4)(4)} = \dfrac{-1}{16}$

(23) $\lim_{x\to 2} \dfrac{8-x^3}{x^2-2x}$

$= \lim_{x\to 2} \dfrac{(2-x)(4+2x+x^2)}{x(x-2)}$

$= \lim_{x\to 2} \dfrac{-1(4+2x+x^2)}{x}$

$= \dfrac{-1(4+4+4)}{2}$

$= -6$

(24) $\lim_{x\to -3^-} \dfrac{4x^2}{9-x^2}$

$\dfrac{+36}{(-\text{sm})} = -\infty$

(25) $\lim_{x\to 3} \dfrac{\frac{1}{x+2} - \frac{1}{5}}{x-3}$

$= \lim_{x\to 3} \dfrac{5-(x+2)}{5(x+2)} \cdot \dfrac{1}{x-3}$

$= \lim_{x\to 3} \dfrac{3-x^{-1}}{5(x+2)} \cdot \dfrac{1}{x-3}$

$= \lim_{x\to 3} \dfrac{-1}{5(x+2)}$

$= \dfrac{-1}{25}$