Pg. 219 # 10, 13, 14, 24-31

10.(a) $f(x) = 2x^3 - 9x^2$ $\qquad -2 \leq x \leq 4$

$f'(x) = 6x^2 - 18x$

(i) Solve $f'(x) = 0 \Rightarrow 6x^2 - 18x = 0$

$6x(x-3) = 0$

$x = 0 \quad x = 3$

(ii) no values where $f'(x)$ is und.

Check by sub. critical numbers and endpts in $f(x)$

$f(-2) = 2(-2)^3 - 9(-2)^2 = -52$

$f(0) = 2(0)^3 - 9(0)^2 = 0$

$f(3) = 2(3)^3 - 9(3)^2 = -27$

$f(4) = 2(4)^3 - 9(4)^2 = -16$

$\therefore$ The maximum value is $f(0) = 0$ and the minimum value is $f(-2) = -52$.

(b) $f(x) = 12x - x^3$ $\qquad -3 \leq x \leq 5$

$f'(x) = 12 - 3x^2$

(i) Solve $f'(x) = 0 \Rightarrow 12 - 3x^2 = 0$

$12 = 3x^2$

$4 = x^2$

$x = 2 \quad x = -2$

(ii) no values where $f'(x)$ is und.

Check: $f(-3) = 12(-3) - (-3)^3 = -9$

$f(-2) = 12(-2) - (-2)^3 = -16$

$f(2) = 12(2) - (2)^3 = 16$

$f(5) = 12(5) - (5)^3 = -65$

$\therefore$ The maximum value is $f(2) = 16$ and the minimum value is $f(5) = -65$.

10. (c) $f(x) = 2x + \frac{18}{x}$ $\qquad 1 \leq x \leq 5$

$f'(x) = 2 - \frac{18}{x^2}$

(i) Solve $f'(x) = 0 \Rightarrow 2 - \frac{18}{x^2} = 0$

$$2 = \frac{18}{x^2}$$
$$2x^2 = 18$$
$$x^2 = 9$$
$$x = 3 \text{ or } \underbrace{x = -3}_{\text{not in domain}}$$

(ii) $f'(x)$ is und. when $x = 0$ (not in given domain)

Check: $f(1) = 2(1) + \frac{18}{(1)} = 20$

$f(3) = 2(3) + \frac{18}{(3)} = 12$

$f(5) = 2(5) + \frac{18}{5} = 13.6$

$\therefore$ The maximum value is $f(1) = 20$ and the minimum value is $f(3) = 12$.

13. $s(t) = 1 + 2t - \frac{8}{t^2+1}$ $\qquad 0 \leq t \leq 2$

$v(t) = 2 + 8(t^2+1)^{-2}(2t)$
$\quad = 2 + 16t(t^2+1)^{-2}$

To determine max/min velocity, need $v'(t)$.

$v'(t) = 16 \cdot (t^2+1)^{-2} + 16t\left[-2(t^2+1)^{-3}(2t)\right]$
$\quad = 16(t^2+1)^{-2} - 64t^2(t^2+1)^{-3}$
$\quad = 16(t^2+1)^{-3}\left[(t^2+1) - 4t^2\right]$
$\quad = \frac{16}{(t^2+1)^3}(-3t^2+1)$

$$v'(t) = \frac{16(-3t^2+1)}{(t^2+1)^3}$$

(i) Solve $v'(t)=0 \Rightarrow \frac{16(-3t^2+1)}{(t^2+1)^3} = 0$

(ii) $v'(t)$ is defined for all values of t in domain

$$-3t^2+1=0$$
$$1 = 3t^2$$
$$\frac{1}{3} = t^2$$
$$t = \frac{1}{\sqrt{3}}$$

Check: $v(0) = 2 + 16(0)[(0)^2+1]^{-2} = 2$

$$v\left(\tfrac{1}{\sqrt{3}}\right) = 2 + 16\left(\tfrac{1}{\sqrt{3}}\right)\left[\left(\tfrac{1}{\sqrt{3}}\right)^2+1\right]^{-2} = 2 + \frac{\left(\frac{16}{\sqrt{3}}\right)}{\left(\frac{16}{9}\right)} = 2+3\sqrt{3} \doteq 7.20$$

$$v(2) = 2 + 16(2)[(2)^2+1]^{-2} = 2 + \frac{32}{25} \doteq 3.28$$

$\therefore$ The maximum velocity is 7.20 m/s when $t = \frac{1}{\sqrt{3}}$ sec.
The minimum velocity is 2 m/s when $t=0$ sec.

14. $C(x) = 625 + 15x + 0.01x^2 \qquad 1 \le x \le 500$

$$U(x) = \frac{C(x)}{x} = \frac{625 + 15x + 0.01x^2}{x}$$

$$U(x) = \frac{625}{x} + 15 + 0.01x$$

To minimize unit cost, $U'(x) = \frac{-625}{x^2} + 0.01$

(i) Solve $U'(x)=0 \implies -\frac{625}{x^2}+0.01=0$

$$0.01=\frac{625}{x^2}$$
$$0.01x^2=625$$
$$x^2=62500$$
$$x=250 \qquad \underbrace{x=-250}_{\text{not in domain}}$$

(ii) $U'(x)$ und when $\underbrace{x=0}_{\text{not in domain}}$

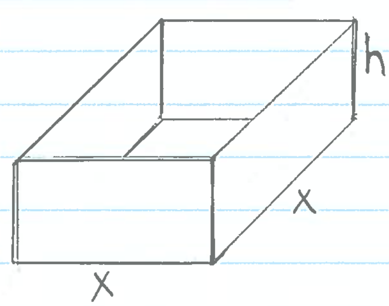
Check: $U(1)=\frac{625}{(1)}+15+0.01(1)=640.01$

$$U(250)=\frac{625}{250}+15+0.01(250)=20$$

$$U(500)=\frac{625}{500}+15+0.01(500)=21.25$$

$\therefore$ The unit cost is minimized when 250 items are manufactured.

24.



Let x be the length and width of the base of the box. Let h be the height. $x, h \geq 5$

Given:
$$V=x^2h=10\,000$$
$$h=\frac{10\,000}{x^2}$$

min $A=x^2+4xh$

sub $h=\frac{10000}{x^2}$

$$A=x^2+4x\left(\frac{10000}{x^2}\right)$$

$$A=x^2+\frac{40000}{x}$$

$A' = 2x - \frac{40000}{x^2}$

(i) Solve $A' = 0 \Rightarrow 2x - \frac{40000}{x^2} = 0$ (ii) A' is und when $x = 0$ (but $x \geq 5$)

$2x = \frac{40000}{x^2}$

$2x^3 = 40000$

$x^3 = 20000$

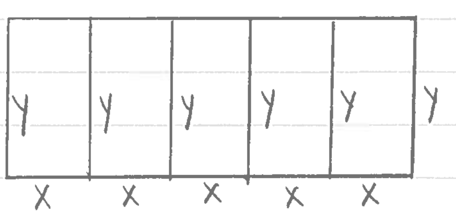
$x \doteq 27.14$

Check: $A(5) = (5)^2 + \frac{40000}{(5)} = 8025 \text{ cm}^2$

$A(27.14) = (27.14)^2 + \frac{40000}{(27.14)} = 2210.4 \text{ cm}^2$

Find h: $h = \frac{10000}{(27.14)^2} \doteq 13.6 \text{ cm}$

∴ To minimize the amount of material, the base of the box should be 27.14 cm x 27.14 cm and the height should be 13.6 cm.

25. 

Let x be the width of each pen. Let y be the length of each pen.

$10 \leq x \leq 240$

given ($x = 240$ when $y = 10$)

Given: $x \cdot y = 2400 \text{ m}^2$

$y = \frac{2400}{x}$

min $P = 10x + 6y$

sub $y = \frac{2400}{x}$

$P = 10x + 6\left(\frac{2400}{x}\right)$

$$P = 10x + \frac{14\,400}{x}$$

$$P' = 10 - \frac{14\,400}{x^2}$$

(i) Solve $P' = 0 \Rightarrow 10 - \frac{14\,400}{x^2} = 0$

$$10 = \frac{14\,400}{x^2}$$

$$10x^2 = 14400$$
$$x^2 = 1440$$
$$x \doteq 37.9 \qquad \underbrace{x \doteq -37.9}_{\text{not in domain}}$$

(ii) P' und when $x=0$, but $x \geq 10$

Check: $P(10) = 10(10) + \frac{14\,400}{10} \doteq 1540$ m

$$P(37.9) = 10(37.9) + \frac{14400}{37.9} \doteq 758.9 \text{ m}$$

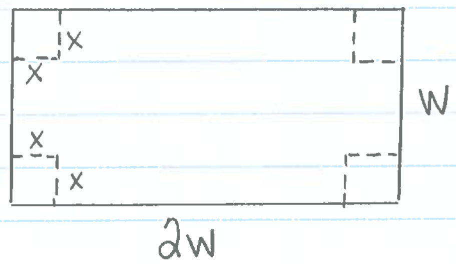
$$P(240) = 10(240) + \frac{14\,400}{240} \doteq 2460 \text{ m}$$

Find y: $y = \frac{2400}{37.9}$

$y \doteq 63.3$

$\therefore$ To minimize the amount of fencing used, the dimensions of each pen should be 37.9 m $\times$ 63.3 m

26.



Let w be the width of the piece of metal. Then the length is $2w$.

Given $A = 800\ dm^2$

$A = 2w(w) = 800$

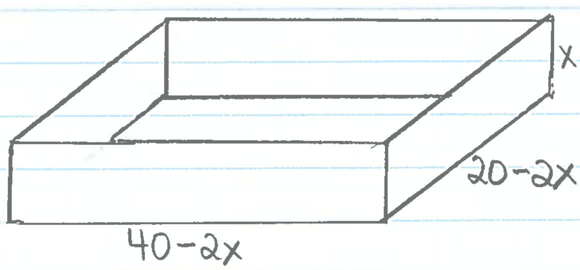
$2w^2 = 800$

$w^2 = 400$

$w = 20\ dm\ (w>0)$

$\therefore l = 2w = 40\ dm$

Let a square be cut from each corner with side length x dm.
Folding up the sides gives a box with the following dimensions



$0 \leq x \leq 10$

↑ since $2x \leq 20$
$x \leq 10$

max $V = (40-2x)(20-2x)x$
$= (800 - 120x + 4x^2)x$
$V = 4x^3 - 120x^2 + 800x$

$V' = 12x^2 - 240x + 800$

(i) Solve $V' = 0$
$\Rightarrow 12x^2 - 240x + 800 = 0$
$4(3x^2 - 60x + 200) = 0$

$x = \dfrac{60 \pm \sqrt{(-60)^2 - 4(3)(200)}}{2(3)}$

$= \dfrac{60 \pm \sqrt{1200}}{6}$

inadmiss. since $x \leq 10$ → $x \doteq 15.8$ or $x \doteq 4.2$

(ii) V' is never und

Check:

$V(0) = 4(0)^3 - 120(0)^2 + 800(0)$
$= 0$

$V(4.2) \doteq 1539.6$

$V(10) = 0$

$l = 40 - 2x = 31.6$ dm
$w = 20 - 2x = 11.6$

$\therefore$ The dimensions of the box are 31.6 dm × 11.6 dm × 4.2 dm

27.

Let r be the radius and let h be the height.

$6 \le h \le 15$ cm

Given $V = 500\ cm^3$

$\pi r^2 h = 500$

$h = \frac{500}{\pi r^2}$

When $h = 6$, $r \doteq 5.2$

$h = 15$, $r \doteq 3.3$

$\therefore\ 3.3 \le r \le 5.2$

min $A = 2\pi r^2 + 2\pi r h$

Sub $h = \frac{500}{\pi r^2}$

$A = 2\pi r^2 + 2\pi r\left(\frac{500}{\pi r^2}\right)$

$A = 2\pi r^2 + \frac{1000}{r}$

$A' = 4\pi r - \frac{1000}{r^2}$

(i) Solve $A' = 0$

$4\pi r - \frac{1000}{r^2} = 0$

$4\pi r = \frac{1000}{r^2}$

$r^3 = \frac{1000}{4\pi}$

$r \doteq 4.3$ cm

(ii) A' is und. when $r = 0$, (not in domain)

Check: $A(3.3) = 2\pi(3.3)^2 + \frac{1000}{(3.3)} \doteq 371.5\ cm^2$

$A(4.3) = 2\pi(4.3)^2 + \frac{1000}{(4.3)} \doteq 348.7\ cm^2 \Leftarrow$ min

$A(5.2) = 2\pi(5.2)^2 + \frac{1000}{(5.2)} \doteq 362.2\ cm^2$

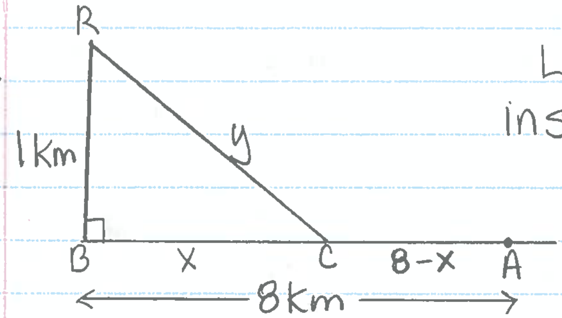
Find h:

$h = \frac{500}{\pi(4.3)^2}$

$\doteq 8.6$ cm

$\therefore$ To minimize the amount of material, $r = 4.3$ cm & $h = 8.6$ cm.

28.



Let the distance the pipe is installed on land be "8−x".
Let y be the distance it is installed under water.
$0 \le x \le 8$ km.

Let the cost on land be \$k per km. Since the cost under water is 60% more, then the cost under water is \$1.6k per km.

minimize cost: C = cost on land + cost under water
$C = k(8-x) + 1.6ky$

From diagram,
$y^2 = x^2 + 1 \Rightarrow$ sub $y = \sqrt{x^2+1}$

$C = k(8-x) + 1.6k\sqrt{x^2+1}$
$= 8k - kx + 1.6k(x^2+1)^{1/2}$

$C' = -k + 0.8k(x^2+1)^{-1/2}(2x)$
$= -k + 1.6kx(x^2+1)^{-1/2}$

(i) Solve $C' = 0$:

$-k + \frac{1.6kx}{\sqrt{x^2+1}} = 0$

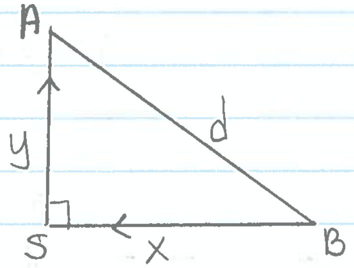
$\frac{1.6kx}{\sqrt{x^2+1}} = k$

$1.6kx = k\sqrt{x^2+1}$
$1.6x = \sqrt{x^2+1}$
$2.56x^2 = x^2+1$
$1.56x^2 = 1$
$x = \sqrt{\frac{1}{1.56}}$
$x \doteq 0.8$ km.
($\because x \ge 0$)

(ii) C' is defined for all x values in domain.

Check:
$C(0) = 8k + 1.6k = 9.6k$
$C(0.8) \doteq 8k - 0.8k + 2.05k \doteq 9.25k$
$C(8) \doteq 8k - 8k + 12.9k \doteq 12.9k$

The cost is minimized when $x = 0.8$ km
$\therefore$ The pipeline should be installed 7.2 km on land, then 1.28 km under water.

29. 

$0 \le t \le 1$

Let y be the distance between the northbound train and the station.
Let x be the distance between the westbound train and the station.
Let t be the time (in hrs) after 10:00.
Let d be the distance between them.

After t hrs, $y = 100t$
Since the westbound train is 120 km from the station at 10:00, and the train travels $120t$ km in t hrs, then $x = 120 - 120t$

From the diagram, $d^2 = (100t)^2 + (120 - 120t)^2$
$= 10000t^2 + 14400 - 28800t + 14400t^2$

$d^2 = 24400t^2 - 28800t + 14400$

Differentiate: $2d\frac{dd}{dt} = 48800t - 28800$

$$\frac{dd}{dt} = \frac{48800t - 28800}{2\sqrt{(100t)^2 + (120-120t)^2}}$$

$$\frac{dd}{dt} = \frac{24400t - 14400}{\sqrt{(100t)^2 + (120-120t)^2}}$$

(i) Solve $\frac{dd}{dt} = 0$

$24400t - 14400 = 0$
$24400t = 14400$
$t = \frac{14400}{24400}$
$t \doteq 0.59$

When $t \doteq 0.59$, $d \doteq 76.8$ km (approx 35.4 minutes after 10:00)

∴ The closest distance between the trains is 76.8 km which occurs at approx 10:35.

(ii) $\frac{dd}{dt}$ is defined for all values of t in domain

30. Let x be the number of CD players sold each month.
Let p be the price per CD player.

Given the store sells 120 CD players at \$100 each
then $(x_1, p_1) = (120, 100)$

Given that a \$2 price increase will result in one less CD player sold, then $(x_2, p_2) = (119, 102)$.

To find $p(x)$: $m = \frac{\Delta p}{\Delta x} = \frac{102-100}{119-120} = \frac{2}{-1} = -2$

$$\frac{-2}{1} = \frac{p-100}{x-120}$$

$$-2x+240 = p-100$$
$$-2x+340 = p$$
$$\therefore\ p(x) = -2x+340$$

Now, $R(x) = x \cdot p(x)$
$= x(-2x+340)$
$= -2x^2+340x$

Given $C(x) = 70x$

$P(x) = R(x) - C(x)$
$= (-2x^2+340x) - 70x$
$P(x) = -2x^2+270x$

To maximize profit, find $P'(x) = -4x+270$

(i) solve $P'(x)=0$
$-4x+270=0$
$270 = 4x$
$x = 67.5$

(ii) $P'(x)$ is never undefined

Check: $P''(x) = -4$
$\therefore$ a max at 67.5

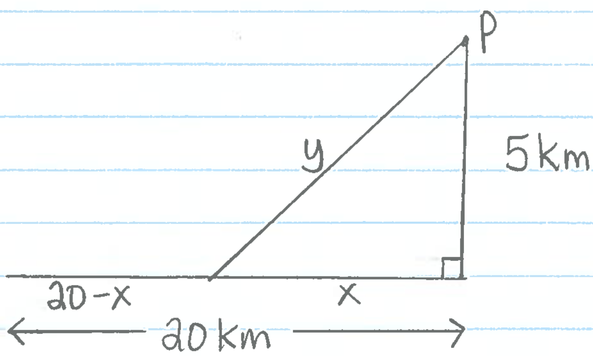
Hilroy

when $x=67.5 \Rightarrow p(67.5) = -2(67.5)+340$
$= \$205.$

$\therefore$ To maximize profit, the store should charge \$205/CD player to sell 67.5 CD players/month.

(Realistically, since they can't sell 0.5 unit, they will sell 67 units if they charge \$206 per unit, or 68 units if they charge \$204 per unit.)

31.



Let "20−x" be the distance the pipeline should be installed on land. Let y be the distance the pipeline should be installed under water.

$0 \le x \le 20$

From the diagram,

$$y^2 = x^2 + 5^2$$
$$y = \sqrt{x^2+25}$$

min Cost = cost on land + cost underwater

$$C = 75000(20-x) + 100000y$$

Sub $y = \sqrt{x^2+25}$

$$C(x) = 1\,500\,000 - 75000x + 100000\sqrt{x^2+25}$$

$$C'(x) = -75000 + 50\,000(x^2+25)^{-1/2}(2x)$$
$$= -75000 + \frac{100\,000x}{\sqrt{x^2+25}}$$

(i) solve $C'(x)=0$

(ii) $C'(x)$ is defined for all x values in domain

$$-75000+\frac{100\,000x}{\sqrt{x^2+25}}=0$$

$$\frac{100\,000x}{\sqrt{x^2+25}}=75\,000$$

$$100\,000x=75\,000\sqrt{x^2+25}$$
$$100x=75\sqrt{x^2+25}$$
$$10000x^2=5625(x^2+25)$$
$$10000x^2=5625x^2+140625$$
$$4375x^2=140625$$
$$x^2=\frac{140625}{4375}$$
$$x\doteq 5.7 \quad (x\geq 0)$$

Check: $C(0)=2\,000\,000$

$C(5.7)\doteq 1830721.60$

$C(20)\doteq 2061\,552.81$

$\therefore$ distance on land $\Rightarrow 20-5.7=14.3$ km
distance under water $\Rightarrow \sqrt{(5.7)^2+25}\doteq 7.6$ km