Pg 214 # 4, 9, 10, 12a

4. $C = 4000 + \frac{h}{15} + \frac{15\,000\,000}{h}$ $\quad 1000 \leq h \leq 20\,000$

$C' = \frac{1}{15} - \frac{15\,000\,000}{h^2}$

* C' und when $h=0$ (but $h>0$)

Solve $C'=0 \Rightarrow \frac{1}{15} - \frac{15\,000\,000}{h^2} = 0$

$\frac{1}{15} = \frac{15\,000\,000}{h^2}$

$h^2 = 225\,000\,000$

$h = 15\,000 \quad (h>0)$

Check: $C(1000) = 4000 + \frac{1000}{15} + \frac{15\,000\,000}{1000} \doteq 19067$

$C(15000) = 4000 + \frac{15000}{15} + \frac{15\,000\,000}{15000} = 6000$

$C(20000) = 4000 + \frac{20000}{15} + \frac{15\,000\,000}{20\,000} \doteq 6083$

$\therefore$ The minimum operating cost is \$6000/hr at a cruising height of 15000 m.

9. Let x be the number of bus riders per day.
Let p be the price per rider.
$0 \leq x \leq 15\,000$

Given:
10 000 riders @ \$20/person $\Rightarrow (x_1, p_1) = (10000, 20)$
\$.50 ↑ in fare reduces number of riders by 200
$\Rightarrow (x_2, p_2) = (9800, 20.50)$

To find $p(x)$: $m = \frac{\Delta p}{\Delta x} = \frac{20.50 - 20}{9800 - 10000} = \frac{0.50}{-200} = \frac{-1}{400}$

$$\frac{-1}{400} = \frac{p - 20}{x - 10000}$$

$$-x + 10000 = 400p - 8000$$
$$-x + 18000 = 400p$$
$$\frac{-1}{400}x + 45 = p \qquad \Rightarrow p(x) = \frac{-1}{400}x + 45$$

Determine $R(x) = x \cdot p(x)$
$$= x\left(\frac{-1}{400}x + 45\right)$$
$$= \frac{-1}{400}x^2 + 45x$$

To maximize revenue, find $R'(x) = \frac{-1}{200}x + 45$

Solve $R'(x) = 0$
$$\frac{-1}{200}x + 45 = 0$$
$$45 = \frac{1}{200}x$$
$$x = 9000$$

* $R'(x)$ is never undefined
* Check (left out)
Find $p(9000) = \frac{-1}{400}(9000) + 45$
$= \$22.50$

∴ To maximize revenue, the fare should be \$22.50 per rider.

10. Let s be the speed of the ship. $\quad T = \frac{D}{s} = \frac{500}{s}$

Let C be the fuel cost

$$C = \text{fuel cost per hour} \times \overbrace{\#\,\text{hrs for trip}}$$
$$= \left(\frac{1}{2}s^3 + 216\right) \times \left(\frac{500}{s}\right)$$
$$= 250s^2 + \frac{108000}{s}$$

$$C' = 500s - \frac{108000}{s^2}$$

Solve $C' = 0 \Rightarrow 500s - \frac{108000}{s^2} = 0$

$$500s = \frac{108000}{s^2}$$
$$500s^3 = 108000$$
$$s^3 = 216$$
$$s = 6$$

C' und when $s = 0$ (but $s > 0$)

Check using Second Derivative Test:

$$C'' = 500 + \frac{216000}{s^3}$$

$$C''(6) = 500 + \frac{216000}{(6)^3} > 0 \quad \therefore \text{ a min at } s = 6$$

$\therefore$ The most economical speed is 6 nautical miles per hour.

12.(a) Let x be the number of cakes sold per week. Let p be the price per cake.

Given they sell 200 cakes at \$10 each
$\Rightarrow (x_1, p_1) = (200, 10)$
Since a price increase of \$0.50 causes a decrease in sales of 7 cakes, $(x_2, p_2) = (193, 10.50)$

To find $p(x)$: $m = \dfrac{\Delta p}{\Delta x} = \dfrac{10.50 - 10}{193 - 200} = \dfrac{0.50}{-7} = \dfrac{-1}{14}$

$$\frac{-1}{14} = \frac{p - 10}{x - 200}$$

$$-x + 200 = 14p - 140$$
$$-x + 340 = 14p \qquad \Rightarrow p(x) = \frac{-1}{14}x + \frac{170}{7}$$

Find $R(x) = x \cdot p(x)$
$$= x\left(\frac{-1}{14}x + \frac{170}{7}\right)$$
$$= \frac{-1}{14}x^2 + \frac{170}{7}x$$

Given $C(x) = 6x$

Find $P(x) = R(x) - C(x)$
$$= \left(\frac{-1}{14}x^2 + \frac{170}{7}x\right) - 6x$$
$$= \frac{-1}{14}x^2 + \frac{128}{7}x$$

$$P'(x) = \frac{-1}{7}x + \frac{128}{7}$$

Solve $P'(x) = 0$
$$\frac{-1}{7}x + \frac{128}{7} = 0$$
$$\frac{128}{7} = \frac{1}{7}x$$
$$x = 128$$

* Check.
$$p(128) = \frac{-1}{14}(128) + \frac{170}{7}$$
$$\doteq 15.14$$

$\therefore$ The optimal sales price to maximize profit is approx \$15 per cake.