Exercise 5.4 (Pg 201)

3. a. $f(x) = x^2 - 4x + 3, 0 \le x \le 3$

$f'(x) = 2x - 4$

Let $2x - 4 = 0$ for max or min

$x = 2$

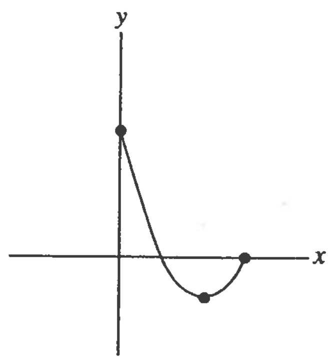
$f(0) = 3$

$f(2) = 4 - 8 + 3 = -1$

$f(3) = 9 - 12 + 3 = 0$

max is 3 at $x = 0$

min is -1 at $x = 2$



c. $f(x) = x^3 - 3x^2, -1 \le x \le 3$

$f'(x) = 3x^2 - 6x$

Let $f'(x) = 0$ for max or min

$3x^2 - 6x = 0$

$3x(x - 2) = 0$

$x = 0$ or $x = 2$

$f(-1) = -1 - 3$

$= -4$

$f(0) = 0$

$f(2) = 8 - 12$

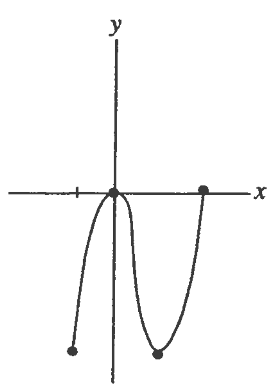
$= -4$

$f(3) = 27 - 27$

$= 0$

min is -4 at $x = -1, 2$

max is 0 at $x = 0, 3$



e. $f(x) = 2x^3 - 3x^2 - 12x + 1, -2 \le x \le 0$

$f'(x) = 6x^2 - 6x - 12$

Let $f'(x) = 0$ for max or min

$6x^2 - 6x - 12 = 0$

$x^2 - x - 2 = 0$

$(x - 2)(x + 1) = 0$

$x = 2$ or $x = -1$

$f(-2) = -16 - 12 + 24 + 1$

$= -3$

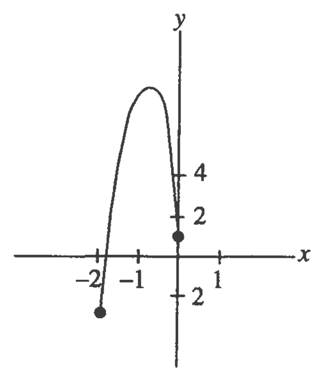
$f(-1) = 8$

$f(0) = 1$

$f(2)$ = not in region

max of 8 at $x = -1$

min of -3 at $x = -2$



4. b. $f(x) = 4\sqrt{x} - x, 2 \le x \le 9$

$f'(x) = 2x^{-\frac{1}{2}} - 1$

Let $f'(x) = 0$ for max or min

$\frac{2}{\sqrt{x}} - 1 = 0$

$x = \sqrt{2}$

$x = 4$

$f(2) = 4\sqrt{2} - 2 \doteq 3.6$

$f(4) = 4\sqrt{4} - 4 = 4$

$f(9) = 4\sqrt{9} - 9 = 3$

min value of 3 when $x = 9$

max value of 4 when $x = 4$

c. $f(x) = \frac{1}{x^2 - 2x + 2}, 0 \le x \le 2$

$f'(x) = -(x^2 - 2x + 2)^{-2}(2x - 2)$

$= -\frac{2x - 2}{(x^2 - 2x + 2)^2}$

Let $f'(x) = 0$ for max or min.

$-\frac{2x - 2}{(x^2 - 2x + 2)} = 0$

$\therefore 2x - 2 = 0$

$x = 1$

$f(0) = \frac{1}{2}, f(1) = 1, f(2) = \frac{1}{2}$

max value of 1 when $x = 1$

min value of $\frac{1}{2}$ when $x = 0, 2$

e. $f(x) = \frac{4x}{x^2 + 1}, -2 \le x \le 4$

$f'(x) = \frac{4(x^2 + 1) - 2x(4x)}{(x^2 + 1)^2}$

$= \frac{-4x^2 + 4}{x^2 + 1}$

Let $f'(x) = 0$ for max or min:

$-4x^2 + 4 = 0$

$x^2 = 1$

$x = \pm 1$

$f(-2) = \frac{-8}{5}$

$f(-1) = \frac{-4}{2}$

$= -2$

$f(1) = \frac{4}{2}$

$= 2$

$f(4) = \frac{16}{17}$

max value of 2 when $x = 1$

min value of -2 when $x = -1$

5. a. $v(t) = \frac{4t^2}{4 + t^3}, t \ge 0$

Interval $1 \le t \le 4$

$v(1) = \frac{4}{5}\ v(4)$

$= \frac{16}{17}$

$v'(t) = \frac{(4 + t^3)(8t) - 4t^2(3t^2)}{(4 + t^3)^2} = 0$

$32t + 8t^4 - 12t^4 = 0$

$-4t(t^3 - 8) = 0$

$t = 0, t = 2$

$v(2) = \frac{16}{12} = \frac{4}{3}$

max velocity is $\frac{4}{3}$ m/s

min velocity is $\frac{4}{5}$ m/s

7. a. $E(v) = \frac{1600v}{v^2 + 6400}\ 0 \le v \le 100$

$E'(v) = \frac{1600(v^2 + 6400) - 1600v(2v)}{(v^2 + 6400)^2}$

Let $E'(N) = 0$ for max or min

$\therefore 1600v^2 + 6400 \times 1600 - 3200v^2 = 0$

$1600v^2 = 6400 \times 1600$

$v = \pm 80$

$E(0) = 0$

$E(80) = 10$

$E(100) = 9.756$

The legal speed limit that maximizes fuel efficiency is 80 km/h.

8. $C(t) = \frac{0.1t}{(t + 3)^2}, 1 \le t \le 6$

$C'(t) = \frac{0(t + 3)^2 - 0.2t(t + 3)}{(t + 3)^4} = 0$

$(t + 3)(0.1t + 0.3 - 0.2t) = 0$

$t = 3$

$C(1) \doteq 0.00625$

$C(3) = 0.0083, C(6) \doteq 0.0074$

The min concentration is at $t = 1$ and the max concentration is at $t = 3$.