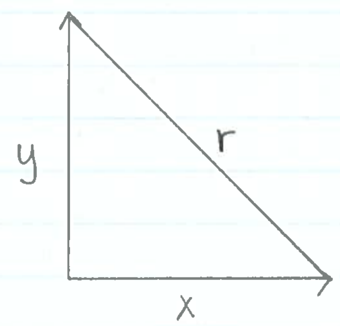
More Related Rates - Solutions

1.



Let x be the distance travelled by the plane travelling east.
Let y be the distance travelled by the plane travelling north.
Let r be the distance between them

Given: $\frac{dx}{dt} = 225$ km/h

$\frac{dy}{dt} = 300$ km/h

After 4 min ($\frac{1}{15}$ hr)

$x = \frac{1}{15}(225) = 15$ km

$y = \frac{1}{15}(300) = 20$ km

Find $\frac{dr}{dt} = ?$ after 4 min.

$r^2 = 15^2 + 20^2$
$r^2 = 625$
$r = 25$

From the diagram,

$x^2 + y^2 = r^2$

Diff wrt "t"

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2r\frac{dr}{dt}$

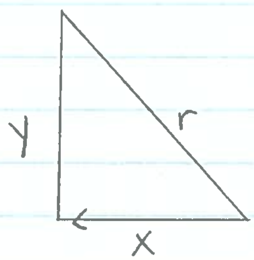
$2(15)(225) + 2(20)(300) = 2(25)\frac{dr}{dt}$

$\frac{18750}{50} = \frac{dr}{dt}$

$\frac{dr}{dt} = 375$ km/h

$\therefore$ After 4 min, the planes are separating at 375 km/h.

2.



Let x be the distance from the passenger car to the railway crossing. Let y be the distance from the locomotive to the railway crossing. Let r be the distance between them.

Given: $\frac{dx}{dt} = -40 \text{ km/h}$ Find: $\frac{dr}{dt} = ?$

$\frac{dy}{dt} = -50 \text{ km/h}$

when $x = 30\text{ m} = .03\text{ km}$

$y = 40\text{ m} = .04\text{ km}$

$r^2 = .03^2 + .04^2$

$r = .05$

By Pythagorean Thm: $x^2 + y^2 = r^2$

Diff. wrt. "t"

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2r\frac{dr}{dt}$$

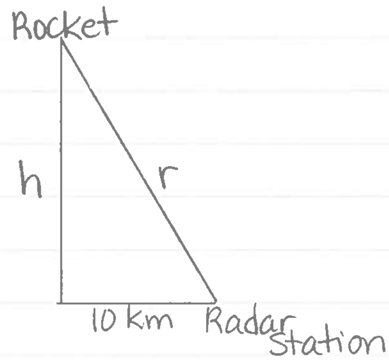
$$2(.03)(-40) + 2(.04)(-50) = 2(.05)\frac{dr}{dt}$$

$$-2.4 - 4 = 0.1\frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{-6.4}{0.1} = -64 \text{ km/h}$$

∴ When the car is 30m from the intersection and the train is 40m from the intersection, the distance between them is decreasing at 64 km/h.

3.



Let h be the height of the rocket above the ground. Let r be the distance between the rocket and the radar station.

Given: $\frac{dr}{dt} = 1000$ km/h   Find $\frac{dh}{dt} = ?$ when $h = 8$ km

$r^2 = 8^2 + 10^2$
$r^2 = 164$
$r = 2\sqrt{41}$

By Pyth. Thm, $10^2 + h^2 = r^2$

Diff wrt "t"

$0 + 2h\frac{dh}{dt} = 2r\frac{dr}{dt}$

When $h=8$: $0 + 2(8)\frac{dh}{dt} = 2(2\sqrt{41})(1000)$

$\frac{dh}{dt} = \frac{4000\sqrt{41}}{16}$

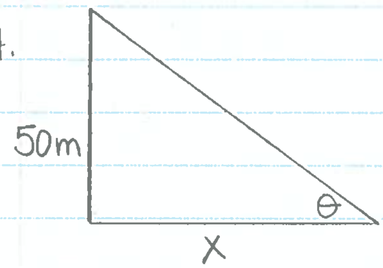
$= 250\sqrt{41}$

$\frac{dh}{dt} \doteq 1600.78$

$\doteq 1601$ km/h

∴ The rocket is rising at a rate of 1601 km/h.

4.



Let θ be the angle of elevation of the sun.
Let x be the length of the shadow of the building.

Given $\frac{d\theta}{dt} = \frac{-1}{4}$ rad/h   Find: $\frac{dx}{dt} = ?$

From diagram, $\cot\theta = \frac{x}{50}$

$$50 \cdot \cot\theta = x$$

Diff wrt "t"

$$50\left(-\csc^2\theta \frac{d\theta}{dt}\right) = \frac{dx}{dt}$$

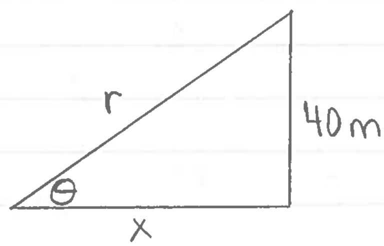
when $\theta = \frac{\pi}{4}$:

$$-50\left(\csc\frac{\pi}{4}\right)^2\left(\frac{-1}{4}\right) = \frac{dx}{dt}$$

$$-50\left(\sqrt{2}\right)^2\left(-\frac{1}{4}\right) = \frac{dx}{dt}$$

$$\frac{dx}{dt} = 25 \text{ m/h}$$

$\therefore$ The shadow of the building is lengthening at 25 m/h.

5. 

Let x be the horizontal distance between the kite and the person on the ground.

Given: $\frac{dx}{dt} = 3$ m/min Find: $\frac{d\theta}{dt} = ?$ when $r = 80$m

From the diagram, $\cot\theta = \frac{x}{40}$

$40 \cdot \cot\theta = x$

Diff. wrt "t"

$40\left(-\csc^2\theta \frac{d\theta}{dt}\right) = \frac{dx}{dt}$

When $r = 80$ m,

$\csc\theta = \frac{80}{40}$

$\csc\theta = 2$

$-40(\csc\theta)^2 \frac{d\theta}{dt} = \frac{dx}{dt}$

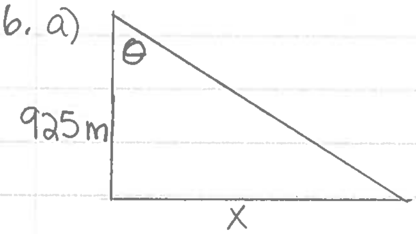
$-40(2)^2 \frac{d\theta}{dt} = 3$

$\frac{d\theta}{dt} = \frac{-3}{160}$ rad/min

$\frac{d\theta}{dt} = -.01875$ rad/min

∴ When 80m of string has been let out, the angle between the string and the ground is decreasing at $\frac{3}{160}$ rad/min.

6\. a)



Let θ be the angle the beacon has revolved from the point closest to shore.
Let x be the distance along the shore.

Given: $\frac{d\theta}{dt} = 2 \text{ rev}/\text{min}$ $= 4\pi \text{ rad}/\text{min}$

Find: $\frac{dx}{dt} = ?$ when $x = 1275$ m

From diagram,

$$\tan\theta = \frac{x}{925}$$

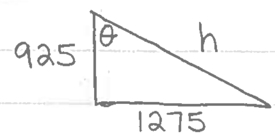
$$925 \cdot \tan\theta = x$$

$$925 \sec^2\theta \frac{d\theta}{dt} = \frac{dx}{dt}$$

$$925\left(\frac{\sqrt{3970}}{37}\right)^2 (4\pi) = \frac{dx}{dt}$$

$$\frac{dx}{dt} \doteq 33708 \text{ m/min}$$

when $x = 1275$ m,



$$h^2 = 925^2 + 1275^2$$
$$= 2481250$$
$$= 25\sqrt{3970}$$

$$\therefore \sec\theta = \frac{25\sqrt{3970}}{925} = \frac{\sqrt{3970}}{37}$$

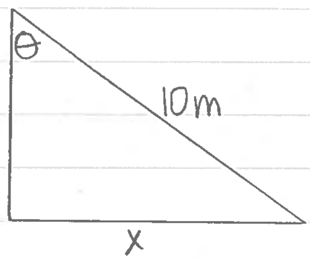
$\therefore$ When the beam of light is 1275 m down the shore, it is sweeping along the shore at 33708 m/min.

b) when the beam is shining at the closest point, $x = 0$, $\theta = 0$

From above, $\frac{dx}{dt} = 925 \sec^2\theta \frac{d\theta}{dt}$ $\because \sec 0 = 1$

$$= 925(1)^2(4\pi)$$
$$\doteq 11624 \text{ m/min}$$

$\therefore$ When the light is shining at the closest point, it is sweeping along the shore at 11 624 m/min.

7. 

Let x be the distance between the base of the ladder and the wall.
Let θ be the angle between the top of the ladder and the wall.

Given: $\frac{dx}{dt} = 2\,m/s$ Find: $\frac{d\theta}{dt} = ?$ when $\theta = \frac{\pi}{4}$

From diagram, $\frac{x}{10} = \sin\theta$

$x = 10 \cdot \sin\theta$

Diff wrt "t"

$\frac{dx}{dt} = 10\cos\theta \frac{d\theta}{dt}$ (sub $\theta = \frac{\pi}{4}$)

$(2) = 10\left(\cos\frac{\pi}{4}\right)\frac{d\theta}{dt}$

$2 = 10\left(\frac{1}{\sqrt{2}}\right)\frac{d\theta}{dt}$

$\frac{d\theta}{dt} = \frac{2\sqrt{2}}{10} = \frac{\sqrt{2}}{5}$ rad/s

∴ The angle between the top of the ladder and the wall is increasing at $\frac{\sqrt{2}}{5}$ rad/s.

* For remaining solutions, see solutions to "Extra Related Rates Review"