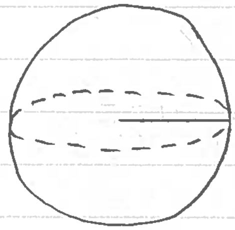
Extra Related Rates Review - Solutions

1. 

Let r be the radius of the balloon and let A be the surface area.

Given $\frac{dd}{dt} = 10 \text{ cm/min}$

Since $d = 2r$,
then $\frac{dr}{dt} = 5 \text{ cm/min}$.

Surface Area of a Sphere:

$$A = 4\pi r^2$$

$$\frac{dA}{dt} = 8\pi r \frac{dr}{dt} \quad \text{Diff. wrt "t"}$$

when $r = 45$ cm

$$\frac{dA}{dt} = 8\pi(45)(5)$$

$$= 1800\pi \text{ cm}^2/\text{min}$$

a) Find $\frac{dA}{dt} = ?$ when $r = 45$ cm

$\therefore$ When the radius is 45cm, the area is increasing at $1800\pi \text{ cm}^2/\text{min}$

b) Find $\frac{dA}{dt} = ?$ when $A = 4\pi \text{ m}^2$

$A = 4\pi r^2$
$4\pi = 4\pi r^2$
$1 = r^2$
$r = 1$ m $(r \geq 0)$
$\therefore r = 100$ cm

From part a)

$$\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$$

when $r = 100$ cm

$$\frac{dA}{dt} = 8\pi(100)(5)$$

$$= 4000\pi \text{ cm}^2/\text{min}$$

$\therefore$ When the surface area is $4\pi \text{ m}^2$, the area is increasing at $4000\pi \text{ cm}^2/\text{min}$.

c) Find $\frac{dA}{dt} = ?$ when $V = \frac{\pi}{6} \text{ m}^3$

$V = \frac{4}{3}\pi r^3$

$\frac{\pi}{6} = \frac{4}{3}\pi r^3$

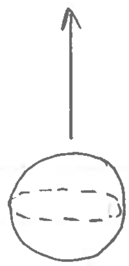
$r^3 = \frac{1}{8} \Rightarrow r = \frac{1}{2}$ m
$= 50$ cm

From part a)

$$\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$$

$$\frac{dA}{dt} = 8\pi(50)(5)$$

$$= 2000\pi \text{ cm}^2/\text{min}$$

2. 

Let V be the volume of the balloon and r be the radius.

Let h be the height above the ground.

Given: $\frac{dh}{dt} = 500\, m/min$

In 2 minutes, h increases 1000m
and r increases 8 cm $\Rightarrow \therefore \frac{dr}{dt} = 4\, cm/min$
$= .04\, m/min$

Find $\frac{dV}{dt} = ?$ when $r = 90$ cm
$= 0.9$ m

Volume of a Sphere: $V = \frac{4}{3}\pi r^3$

Diff wrt "t"

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

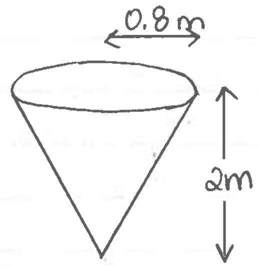
When $r = 0.9$m

$$\frac{dV}{dt} = 4\pi (0.9)^2 (.04)$$

$$= 0.1296\pi \ m^3/min$$

$\therefore$ At the instant the radius is 90 cm, the volume is increasing at $0.1296\pi\ m^3/min$.

3.

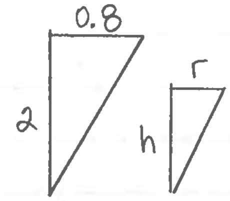


Let h be the height of the water in the tank. Let r be the radius at the top of the water.
Let V be the volume.

Given: $\frac{dV}{dt} = -2\,m^3/min$

Find: $\frac{dh}{dt} = ?$

$V = \frac{1}{3}\pi r^2 h$



From diagram,

$\frac{r}{h} = \frac{0.8}{2}$

$r = 0.4h$

$r = \frac{2}{5}h$

Sub $r = \frac{2}{5}h$

$V = \frac{1}{3}\pi\left(\frac{2}{5}h\right)^2 h$

$V = \frac{4\pi}{75}h^3$

Diff. wrt "t"

$\frac{dV}{dt} = \frac{4\pi}{25}h^2\frac{dh}{dt}$

a) when $h = 1.5m$,
$= \frac{3}{2}\,m$

$-2 = \frac{4\pi}{25}\left(\frac{3}{2}\right)^2\frac{dh}{dt}$

$-2 = \frac{9\pi}{25}\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{-50}{9\pi} \doteq -1.77\,m/min$

∴ When the water level is 1.5m, the level is decreasing at 1.77 m/min.

b) When the tank is <u>full</u>, $V = \frac{1}{3}\pi(0.8)^2(2)$
$\doteq 1.34\,m^3$

∴ half-full, $V = 0.67\,m^3$

Find h when $V = 0.67m^3$

$V = \frac{4\pi}{75}h^3$

$0.67 = \frac{4\pi}{75}h^3$

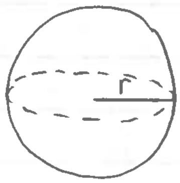
$h^3 = 4 \Rightarrow h \doteq 1.59$

From above, $\frac{dV}{dt} = \frac{4\pi}{25}h^2\frac{dh}{dt}$

$-2 = \frac{4\pi}{25}(1.59)^2\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{-2(25)}{4\pi(1.59)^2} \doteq -1.58$ m/min

∴ When the tank is half full, the level is decreasing at 1.58 m/min

4. 

Let V be the volume of the hailstone and let r be the radius. Let A be the area.

Given: $\frac{dV}{dt} = 1\,mm^3/min$

Find: $\frac{dr}{dt} = ?$

when $A = 4\pi\,cm^2$

SA of sphere $\rightarrow$ $A = 4\pi r^2$

$4\pi = 4\pi r^2$

$r^2 = 1$

$r = 1\,cm$

$= 10\,mm$

Volume of a sphere:

$V = \frac{4}{3}\pi r^3$

Diff wrt "t"

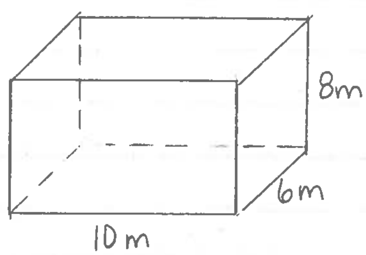
$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

When $A = 4\pi cm^2 \Rightarrow r = 10\,mm$

$(1) = 4\pi(10)^2 \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{1}{400\pi}$ mm/min

$\therefore$ When the surface area is $4\pi\,cm^2$, the radius is increasing at $\frac{1}{400\pi}$ mm/min.

5. 

Let h be the height of the water level in the tank. Let V be the volume.

Given: $\frac{dh}{dt} = 6\,cm/min$

$= .06\,m/min$

Find: $\frac{dV}{dt} = ?$

$V = l \cdot w \cdot h$

$V = (10)(6)h$

$V = 60h$

$l = 10$ & $w = 6$ are <u>constant</u>

Diff wrt "t"

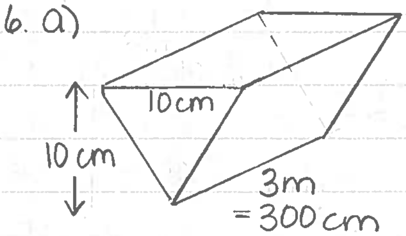
$\frac{dV}{dt} = 60\frac{dh}{dt}$

$\frac{dV}{dt} = 60(.06)$

$= 3.6\,m^3/min$

$\therefore$ Water is flowing in to the tank at the rate $3.6\,m^3/min$.

6. a)



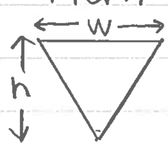
Let V be the volume of water in the trough. Let h be the height of the water in the trough and let w be the width

Given: $\frac{dh}{dt} = 1$ cm/min

Find: $\frac{dV}{dt} = ?$

when $h = 5$ cm

From diagram,



$w = h$

$V = (\text{Area of } \triangledown) \times \text{Length}$

$= \left(\frac{1}{2} wh\right) \times 300$

*L=300 is constant

Sub $w = h$

$V = 150h^2$

Diff. wrt "t"

$\frac{dV}{dt} = 300h\frac{dh}{dt}$

When $h = 5$ cm,

$\frac{dV}{dt} = 300(5)(1)$

$= 1500 \text{ cm}^3/\text{min}$

$\therefore$ When the depth is 5cm, the volume is increasing at 1500 cm³/min.

b) Given: $\frac{dV}{dt} = .06 \text{ m}^3/\text{min}$

$= 60000 \text{ cm}^3/\text{min}$

Find: $\frac{dh}{dt} = ?$

when $h = 1$ cm

From above,

$\frac{dV}{dt} = 300h\frac{dh}{dt}$

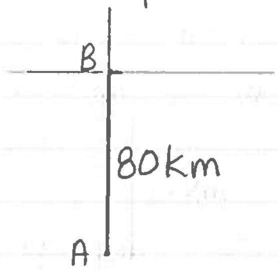
$60000 = 300(1)\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{60000}{300}$

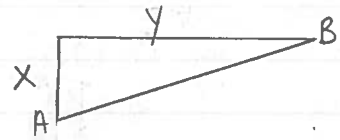
$= 200$ cm/min

$= 2$ m/min

7. 1:00 pm



3:00pm



Let y be the distance travelled by ship B.
Let (80−x) be the distance travelled by ship A.
Let r be the distance between them.

Given: $\frac{dx}{dt} = -30$ km/h

$\frac{dy}{dt} = 40$ km/h

Find: $\frac{dr}{dt}$ after 2h.

At 3:00 pm, $y = 40(2) = 80$

$x = 80 - 30(2) = 20$

$r^2 = 20^2 + 80^2$

$r^2 = 6800$

$r = 20\sqrt{17}$ $(r \geq 0)$

By pythagorean theorem,

$$x^2 + y^2 = r^2$$

Diff wrt "t"

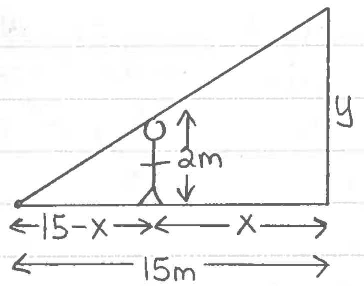
$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2r\frac{dr}{dt}$$

$$2(20)(-30) + 2(80)(40) = 2(20\sqrt{17})\frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{5200}{40\sqrt{17}} \doteq 31.5 \text{ km/h}$$

∴ At 3:00 pm the ships are separating at the rate of 31.5 km/h.

8.



Let x be the distance between the woman and the wall.
Let y be the length of her shadow on the wall.

Given: $\frac{dx}{dt} = 1.1\text{ m/s}$ Find: $\frac{dy}{dt} = ?$ when $x = 3\text{m}$

By similar triangles,

$$\frac{y}{2} = \frac{15}{15-x}$$

$$y = 30(15-x)^{-1}$$

Diff. wrt "t"

$$\frac{dy}{dt} = -30(15-x)^{-2}(-1)\frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{30}{(15-x)^2}\frac{dx}{dt}$$

when $x = 3\text{m}$,

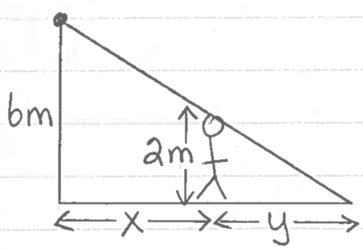
$$\frac{dy}{dt} = \frac{30}{(15-3)^2}\left(\frac{11}{10}\right)$$

$$= \frac{3(11)}{144} = \frac{11}{48}\text{ m/s}$$

$$\doteq 0.23\text{ m/s}$$

∴ When the woman is 3m from the wall, her shadow is lengthening at $\frac{11}{48}$ m/s.

9.



Let x be the distance between the streetlight and the woman.
Let y be the length of her shadow on the ground.

Given: $\frac{dx}{dt} = 1.5\text{ m/s}$

Find: $\frac{dy}{dt} = ?$

$$\frac{6}{x+y} = \frac{2}{y} \quad \text{(By similar triangles)}$$

$$6y = 2x + 2y$$

$$4y = 2x$$

$$y = \frac{1}{2}x$$

Diff wrt "t"

$$\frac{dy}{dt} = \left(\frac{1}{2}\right)\frac{dx}{dt}$$

a) when $x = 3\text{m}$,

$$\frac{dy}{dt} = \left(\frac{1}{2}\right)(1.5)$$

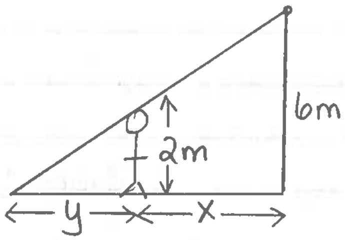
$$= 0.75\text{ m/s}$$

b) when $x = 30\text{m}$,

$$\frac{dy}{dt} = \left(\frac{1}{2}\right)\left(\frac{3}{2}\right)$$

$$= 0.75\text{ m/s}$$

∴ When the woman is 3m (or 30m) from the light, her shadow is lengthening at 0.75 m/s

10. 

Let x be the distance between the man and the streetlight.
Let y be the length of his shadow on the ground.

Given: $\frac{dx}{dt} = -1 m/s$

Find: $\frac{dy}{dt} = ?$

By similar triangles,

$$\frac{2}{y} = \frac{6}{x+y}$$
$$2x+2y = 6y$$
$$4y = 2x$$
$$y = \frac{1}{2}x$$

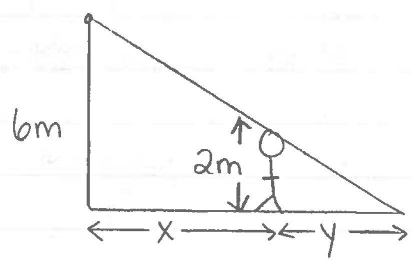
Diff. wrt "t"

$$\frac{dy}{dt} = \frac{1}{2}\frac{dx}{dt}$$

When $\frac{dx}{dt} = -1 m/s$

$$\frac{dy}{dt} = \frac{1}{2}(-1)$$
$$= -0.5 m/s$$

∴ The length of his shadow is decreasing by 0.5 m/s.

11. 

Let x be the distance between the woman and the streetlight.
Let y be the length of her shadow on the ground.

Given: $\frac{dx}{dt} = 80 m/min$

Find: $\frac{dy}{dt} = ?$

when $x = 8 m$

By similar triangles,

$$\frac{2}{y} = \frac{6}{x+y}$$
$$2x+2y = 6y$$
$$4y = 2x$$
$$y = \frac{1}{2}x$$

Diff. wrt "t"

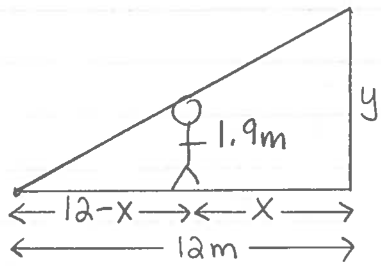
$$\frac{dy}{dt} = \frac{1}{2}\frac{dx}{dt}$$

when $x = 8 m$,

$$\frac{dy}{dt} = \frac{1}{2}(80)$$
$$= 40 m/min$$

∴ Her shadow is increasing in length at the rate 40 m/min.

12.



Let x be the distance between the man and the wall. Let y be the length of his shadow on the wall.

Given: $\frac{dx}{dt} = -1.2\,m/s$

Find: $\frac{dy}{dt} = ?$

when $x = 4m$

By similar triangles,

$$\frac{y}{12} = \frac{1.9}{12-x}$$

$$y = 22.8(12-x)^{-1}$$

Diff. wrt "t"

$$\frac{dy}{dt} = -22.8(12-x)^{-2}(-1)\frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{22.8}{(12-x)^2}\frac{dx}{dt}$$

when $x = 4m$,

$$\frac{dy}{dt} = \frac{22.8}{(12-4)^2}(-1.2)$$

$$= \frac{-171}{400} = -0.4275\ m/s$$

$\therefore$ When he is 4m from the wall, the length of his shadow is decreasing at 0.4275 m/s.