Pg 404 #3, 6, 7

3. (a) $y = \cos x + \sin x \qquad 0 \le x \le 2\pi$

$$\frac{dy}{dx} = -\sin x + \cos x$$

(i) Solve $\frac{dy}{dx} = 0 \quad \Rightarrow -\sin x + \cos x = 0$

$$\cos x = \sin x$$
$$1 = \tan x$$
$$x = \frac{\pi}{4}, \frac{5\pi}{4}$$

(ii) there are no values where $\frac{dy}{dx}$ is undefined.

Check: Sub critical numbers and endpts into $f(x)$

$f(0) = \cos(0) + \sin(0) = 1$

$f(\pi/4) = \cos(\pi/4) + \sin(\pi/4)$

$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$

$= \frac{2}{\sqrt{2}} = \sqrt{2}$

$f\left(\frac{5\pi}{4}\right) = \cos\left(\frac{5\pi}{4}\right) + \sin\left(\frac{5\pi}{4}\right)$

$= \left(\frac{-1}{\sqrt{2}}\right) + \left(\frac{-1}{\sqrt{2}}\right)$

$= \frac{-2}{\sqrt{2}} = -\sqrt{2}$

$\therefore$ The absolute max value is $f(\pi/4) = \sqrt{2}$ and the absolute min value is $f\left(\frac{5\pi}{4}\right) = -\sqrt{2}$

$f(2\pi) = \cos(2\pi) + \sin(2\pi)$

$= 1 + 0$

$= 1$

3 (b) $y = x + 2\cos x \qquad -\pi \leq x \leq \pi$

$\frac{dy}{dx} = 1 - 2\sin x$

(i) Solve $\frac{dy}{dx} = 0 \Rightarrow 1 - 2\sin x = 0$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

(ii) There are no values where $\frac{dy}{dx}$ is undefined.

Check: Sub. critical numbers and endpts. in $f(x)$

$f(-\pi) = (-\pi) + 2\cos(-\pi)$
$= -\pi + 2(-1)$
$= -\pi - 2 \quad \doteq -5.14$

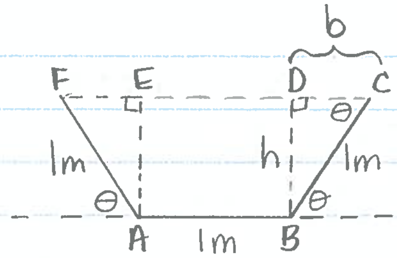
$f(\frac{\pi}{6}) = (\frac{\pi}{6}) + 2\cos(\frac{\pi}{6})$
$= \frac{\pi}{6} + 2(\frac{\sqrt{3}}{2})$
$= \frac{\pi}{6} + \sqrt{3} \quad \doteq 2.26$

$f(\frac{5\pi}{6}) = (\frac{5\pi}{6}) + 2\cos(\frac{5\pi}{6})$
$= \frac{5\pi}{6} + 2(-\frac{\sqrt{3}}{2})$
$= \frac{5\pi}{6} - \sqrt{3} \quad \doteq 0.89$

$\therefore$ The absolute max value is $f(\frac{\pi}{6}) \doteq 2.26$ and the absolute min value is $f(-\pi) = -5.14$

$f(\pi) = (\pi) + 2\cos(\pi)$
$= \pi + 2(-1)$
$= \pi - 2 \quad \doteq 1.14$

6.



Let θ be the angle that the metal is bent up from the horizontal.

$0 \le \theta \le \frac{\pi}{2}$

from diagram

$\sin\theta = \frac{h}{1}$

$\therefore \sin\theta = h$

$\cos\theta = \frac{b}{1} = b$

max cross-sectional area

$A = \text{Area} \square + 2\,\text{Area} \triangledown$

$= 1(h) + \cancel{2}(\frac{1}{\cancel{2}}bh)$

$= h + bh$

Sub $h = \sin\theta$, $b = \cos\theta$

$A = \sin\theta + \cos\theta\sin\theta$

$A' = \cos\theta + [-\sin\theta\sin\theta + \cos\theta\cos\theta]$

$= \cos\theta - \sin^2\theta + \cos^2\theta$

(i) Solve $A' = 0 \Rightarrow \cos^2\theta - \sin^2\theta + \cos\theta = 0$

$\cos^2\theta - (1 - \cos^2\theta) + \cos\theta = 0$ $\quad \because \sin^2\theta = 1 - \cos^2\theta$

$\cos^2\theta - 1 + \cos^2\theta + \cos\theta = 0$

$2\cos^2\theta + \cos\theta - 1 = 0$

$(2\cos\theta - 1)(\cos\theta + 1) = 0$

$\cos\theta = \frac{1}{2}$ $\qquad \underbrace{\cos\theta = -1}_{\text{no sol}^{\underline{n}} \text{ in given domain}}$

$\theta = \frac{\pi}{3}$

(ii) A' is defined for all values of x in the domain

Check: $A(0) = \sin(0) + \cos(0)\sin(0) = 0$

$A(\frac{\pi}{3}) = \sin(\frac{\pi}{3}) + \cos(\frac{\pi}{3})\sin(\frac{\pi}{3})$

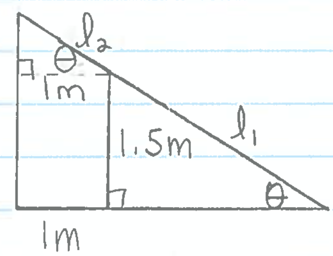
$= \frac{\sqrt{3}}{2} + \frac{1}{2}\cdot\frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{4} \doteq 1.30$

$A(\frac{\pi}{2}) = \sin(\frac{\pi}{2}) + \cos(\frac{\pi}{2})\sin(\frac{\pi}{2})$

$= 1 + 0(1)$

$= 1$

$\therefore$ The max area occurs when $\theta = \frac{\pi}{3}$ radians.

7. 

Let θ be the angle between the ladder and the ground. $(0 < \theta < \frac{\pi}{2})$
Let $l = l_1 + l_2$ be the length of the ladder.

From the diagram, $\csc\theta = \frac{l_1}{1.5}$ $\qquad \sec\theta = \frac{l_2}{1}$

$l_1 = 1.5\csc\theta \qquad l_2 = \sec\theta$

$l = l_1 + l_2$
$l = 1.5\csc\theta + \sec\theta$

$\frac{dl}{d\theta} = -1.5\csc\theta\cot\theta + \sec\theta\tan\theta$

(i) Solve $\frac{dl}{d\theta} = 0 \Rightarrow -1.5\csc\theta\cot\theta + \sec\theta\tan\theta = 0$

$$-1.5\frac{1}{\sin\theta}\cdot\frac{\cos\theta}{\sin\theta} + \frac{1}{\cos\theta}\cdot\frac{\sin\theta}{\cos\theta} = 0$$

$$\frac{-1.5\cos\theta}{\sin^2\theta} + \frac{\sin\theta}{\cos^2\theta} = 0$$

$$\frac{\sin\theta}{\cos^2\theta} = \frac{1.5\cos\theta}{\sin^2\theta}$$

$$\frac{\sin^3\theta}{\cos^3\theta} = \frac{1.5\cos^3\theta}{\cos^3\theta}$$

$$\tan^3\theta = 1.5$$

$$\tan\theta = \sqrt[3]{1.5}$$

$$\tan\theta \doteq 1.144714243$$

$$\theta \doteq 0.853 \text{ rad}$$

(ii) $\frac{dl}{d\theta}$ is und if $\sin\theta = 0$ or $\cos\theta = 0$

$\theta = 0 \qquad \theta = \frac{\pi}{2}$

*not part of domain

Determine $l = 1.5\csc(0.853) + \sec(0.853)$
$= 3.5$ m

*note ⇒ answer in back of text is incorrect *