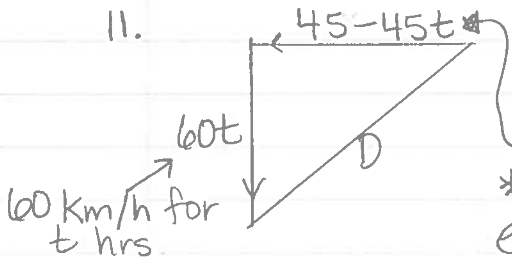
Pg 208 #11 ; Pg 214 #5,7,8

11.



Let t be the time (in hrs) after 10:00 when the two trains are closest together. Let D be the distance between them.

*Note: The westbound train is 45 km east of the station at 10:00. In t hrs it travels $45t$ km, so the distance between the train and the station is $45-45t$

$$\min D^2 = (60t)^2 + (45-45t)^2$$
$$= 3600t^2 + 2025 - 4050t + 2025t^2$$
$$D^2 = 5625t^2 - 4050t + 2025 \qquad 0 \le t \le 1$$
$$2D\frac{dD}{dt} = 11250t - 4050 \qquad \frac{dD}{dt} = \frac{11250t - 4050}{2D}$$

For minimum, $\frac{dD}{dt} = 0$ $\qquad$ $D'(t)$ is defined $\forall$ values of t.

$$11250t - 4050 = 0$$
$$11250t = 4050$$
$$t = 0.36$$

Check: $D(0) = (60\times0)^2 + (45-45\times0)^2 = 2025$ km

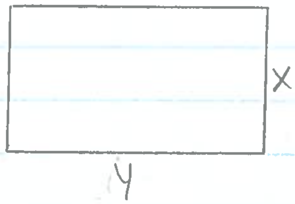
$D(0.36) = (60\times0.36)^2 + (45-45\times0.36)^2 = 1296$ km

$D(1) = (60\times1)^2 + (45-45\times1)^2 = 3600$ km

$\therefore$ The trains were closest together 0.36 hrs after 10:00 (ie. at approx 10:22)

Pg. 214

5.



Let x be the width of the field and let y be the length.

max $A = x \cdot y$ given $C = 6(2x) + 9(2y) = 9000$

$12x + 18y = 9000$

$18y = 9000 - 12x$

$y = 500 - \frac{2}{3}x$

Sub $y = 500 - \frac{2}{3}x$

$A = x\left(500 - \frac{2}{3}x\right)$

$A = 500x - \frac{2}{3}x^2 \quad (0 \leq x \leq 750)$

$A' = 500 - \frac{4}{3}x$

For maximum area, $A'(x) = 0$

$500 - \frac{4}{3}x = 0$

$A'(x)$ is defined $\forall$ values of x

$500 = \frac{4}{3}x$

$x = 375$

Check:

$A(0) = 500(0) - \frac{2}{3}(0)^2 = 0$

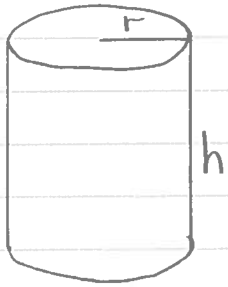
$A(375) = 500(375) - \frac{2}{3}(375)^2 = 93750$ (max)

$A(750) = 500(750) - \frac{2}{3}(750)^2 = 0$

Find y: $y = 500 - \frac{2}{3}(375) = 250$

$\therefore$ The rectangular lot with maximum area has dimensions 250 m x 375 m

7.



Let r be the radius of the can and let h be the height. $r, h > 0$

Let \$$k$ be the cost (per cm^2) for the walls of the can. Since the end pieces are twice as expensive, the cost is \$$2k$ per cm^2

Given:

$V = 1000\ cm^3$

$\pi r^2 h = 1000$

$h = \dfrac{1000}{\pi r^2}$

min Cost = cost of walls + cost of ends

$$C(x) = k(2\pi r h) + 2k(2\pi r^2)$$
$$= 2k\pi r h + 4k\pi r^2$$
$$= 2k\pi r\left(\frac{1000}{\pi r^2}\right) + 4k\pi r^2$$
$$C(x) = \frac{2000k}{r} + 4k\pi r^2$$

$$C'(x) = -\frac{2000k}{r^2} + 8k\pi r$$

(i) Solve $C'(x) = 0$:

$$-\frac{2000k}{r^2} + 8k\pi r = 0$$
$$8k\pi r = \frac{2000k}{r^2}$$
$$8k\pi r^3 = 2000k$$
$$r^3 = \frac{250}{\pi}$$
$$r \doteq 4.3\ cm$$

(ii) $C'(x)$ und when $x = 0$ (but $x > 0$)

Check using Second Derivative Test:

$$C''(x) = \frac{4000k}{r^3} + 4k\pi$$
$$C''(4.3) = \frac{4000k}{(4.3)^3} + 4k\pi > 0$$
(given $k > 0$)

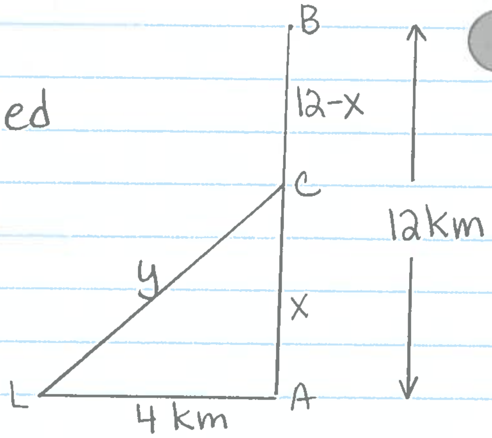
$\therefore$ min at $r = 4.3$ cm

$$h = \frac{1000}{\pi(4.3)^2} = 17.2\ cm.$$

$\therefore$ The cost is minimized when the radius is 4.3 cm and the height is 17.2 cm.

8. Let "12-x" be the distance the power cable will be installed on land. $0 \le x \le 12$

Let y be the distance the power cable will be installed under water.



From diagram, $y^2 = x^2 + 4^2$

$y = \sqrt{x^2+16}$

min Cost = cost on land + cost under water

$C(x = 2000(12-x) + 6000(y)$

sub $y = \sqrt{x^2+16}$

$$C(x) = 24000 - 2000x + 6000\sqrt{x^2+16}$$

$$C'(x) = -2000 + 3000(x^2+16)^{-\frac{1}{2}}(2x)$$
$$= -2000 + \frac{6000x}{\sqrt{x^2+16}}$$

(i) Solve $C'(x) = 0$

$$-2000 + \frac{6000x}{\sqrt{x^2+16}} = 0$$

$$\frac{6000x}{\sqrt{x^2+16}} = 2000$$

$$6000x = 2000\sqrt{x^2+16}$$
$$3x = \sqrt{x^2+16}$$
$$9x^2 = x^2+16$$
$$8x^2 = 16$$
$$x^2 = 2$$
$$x \doteq 1.4 \quad (x \ge 0)$$

(ii) $C'(x)$ is defined for all x values in domain

Check:

$C(0) = 48\,000$

$C(1.4) \doteq 46627.54$

$C(12) = 75894.66$

$\because 12 - x = 10.6$

The cable should be run to a point C located 10.6 km south of the power source.