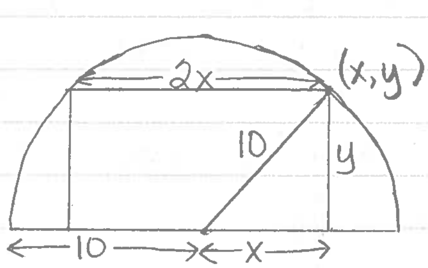
Pg. 206 #6-8, 10

6.



Let the length of the rectangle be $2x$ and let the width be y.

From the equation for the semi-circle, $x^2+y^2=100$

$$y^2=100-x^2$$
$$y=\sqrt{100-x^2}\ ,\ y\geq 0$$

max $A=2x\cdot y$

sub $y=\sqrt{100-x^2}$

$$A=2x\sqrt{100-x^2} \qquad (0\leq x\leq 10)$$

$$\begin{aligned} A' &= 2\sqrt{100-x^2}+2x\left[\tfrac{1}{2}(100-x^2)^{-1/2}(-2x)\right] \\ &= 2(100-x^2)^{1/2}-2x^2(100-x^2)^{-1/2} \\ &= 2(100-x^2)^{-1/2}\left[(100-x^2)-x^2\right] \\ &= 2(100-x^2)^{-1/2}(100-2x^2) \\ &= \frac{4(50-x^2)}{\sqrt{100-x^2}} \end{aligned}$$

For critical numbers,

(i) $A'=\dfrac{4(50-x^2)}{\sqrt{100-x^2}}=0$

$$50-x^2=0$$
$$50=x^2$$
$$x=\sqrt{50}=5\sqrt{2}$$

(ii) A' is undefined if

$$100-x^2=0$$
$$x^2=100$$
$$x=\pm 10 \qquad (x\geq 0)$$

Check: $A(0)=2(0)\sqrt{100-0^2}=0$

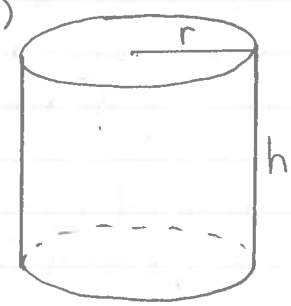
$A(5\sqrt{2})=2(5\sqrt{2})\sqrt{100-(5\sqrt{2})^2}=100$

$A(10)=2(10)\sqrt{100-(10)^2}=0$

$\because l=2x$
$=2(5\sqrt{2})$
$=10\sqrt{2}$

$\therefore$ The largest possible area is $100\ cm^2$ when the length is $10\sqrt{2}$ cm.

7. a)



Let r be the radius of the cylinder and let h be the height.

Given
$V = \pi r^2 h = 1000$
$h = \frac{1000}{\pi r^2}$

min $SA = 2\pi r^2 + 2\pi r h$
sub $h = \frac{1000}{\pi r^2}$
$A = 2\pi r^2 + 2\pi r\left(\frac{1000}{\pi r^2}\right)$
$= 2\pi r^2 + \frac{2000}{r}$, $r \geq 3$ (given)
$A' = 4\pi r - \frac{2000}{r^2}$

For critical numbers,
(i) $A' = 4\pi r - \frac{2000}{r^2} = 0$
$4\pi r = \frac{2000}{r^2}$
$4\pi r^3 = 2000$
$r^3 = \frac{2000}{4\pi}$
$r = \sqrt[3]{\frac{500}{\pi}}$
$r \doteq 5.42$

(ii) A' is undefined when $r = 0$ (not part of domain)

Find h: $h = \frac{1000}{\pi(5.42)^2}$
$\doteq 10.84$

Check using 2nd derivative:

$A'' = 4\pi + \frac{4000}{r^3}$

$A''(5.42) = 4\pi + \frac{4000}{(5.42)^3} > 0$

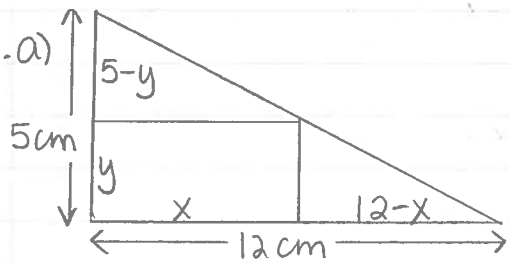
∴ a min occurs at $r = 5.42$

∴ The amount of tin required is a minimum when the radius is 5.4 cm and the height is 10.8 cm.

b) $\frac{h}{d} = \frac{10.84}{2(5.42)} = \frac{10.84}{10.84}$

The ratio $h:d = 1:1$

8. a)



Let x represent the length of the rectangle. and let y represent the width.

By similar triangles,

$$\frac{5}{12} = \frac{y}{12-x}$$

$$12y = 5(12-x)$$

$$12y = 60-5x$$

$$y = 5 - \frac{5}{12}x$$

max $A = x \cdot y$

sub. $y = 5 - \frac{5}{12}x$

$$A = x\left(5 - \frac{5}{12}x\right)$$

$$= 5x - \frac{5}{12}x^2 \quad (0 \le x \le 12)$$

$$A' = 5 - \frac{5}{6}x$$

For critical numbers

(i) $A' = 5 - \frac{5}{6}x = 0$

$$5 = \frac{5}{6}x$$

$$30 = 5x$$

$$x = 6$$

(ii) A' is defined $\forall$ x values in the domain.

Check: $A(0) = 5(0) - \frac{5}{12}(0)^2 = 0$

$A(6) = 5(6) - \frac{5}{12}(6)^2 = \underline{15}$

$A(12) = 5(12) - \frac{5}{12}(12)^2 = 0$

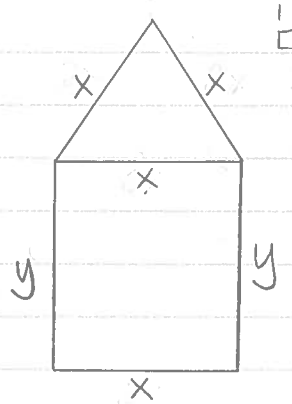
Find y:

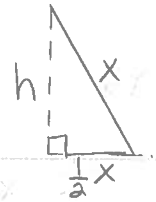
$$y = 5 - \frac{5}{12}(6)$$

$$= 2.5$$

$\therefore$ The largest possible area is 15 cm^2 when the length is 6cm and the width is 2.5 cm

10.





Let the width of the window be x.
Let y be the height of the rectangular part of the window.

$$\max \quad A = \text{area of } \Delta + \text{area of } \square$$
$$= \frac{b \cdot h}{2} + xy$$

Given

$$4x + 2y = 6$$
$$2y = 6 - 4x$$
$$y = 3 - 2x$$

From diagram, $b = x$

$$h^2 = x^2 - \left(\tfrac{1}{2}x\right)^2$$
$$= x^2 - \tfrac{1}{4}x^2$$
$$h^2 = \tfrac{3}{4}x^2 \Rightarrow h = \tfrac{\sqrt{3}}{2}x$$

Sub $y = 3 - 2x$, $b = x$ & $h = \frac{\sqrt{3}}{2}x$:

$$A = \frac{1}{2}(x)\left(\frac{\sqrt{3}}{2}x\right) + x(3 - 2x)$$
$$= \frac{\sqrt{3}}{4}x^2 + 3x - 2x^2 \qquad (0 \leq x \leq 1.5)$$
$$A' = \frac{\sqrt{3}}{2}x + 3 - 4x$$

For critical numbers,

(i) $A' = \frac{\sqrt{3}}{2}x + 3 - 4x = 0$

$$x\left(\frac{\sqrt{3}}{2} - 4\right) = -3$$
$$x = \frac{-3}{\left(\frac{\sqrt{3}}{2} - 4\right)}$$
$$x \doteq 0.96 \text{ m}$$

(ii) A' is defined $\forall$ x values in the domain

Check:

$$A(0) = \frac{\sqrt{3}}{4}(0)^2 + 3(0) - 2(0)^2 = 0$$
$$A(0.96) = \frac{\sqrt{3}}{4}(0.96)^2 + 3(0.96) - 2(0.96)^2$$
$$\doteq 1.436 \text{ m}^2$$
$$A(1.5) = \frac{\sqrt{3}}{4}(1.5)^2 + 3(1.5) - 2(1.5)^2$$
$$\doteq 0.974 \text{ m}^2$$

$\therefore$ The maximum area is approx. 1.44 m^2 when the width is 0.96 m and the height of the rectangle is 1.08 m

$$h = 3 - 2(0.96) = 1.08 \text{ m}$$