Pg 201 #6, 7

6. $C(t) = 30t^2 - 240t + 500$ where C is the concentration of bacteria per cm^3 and t is the time, in days, since the treatment $0 \leq t \leq 7$

$C'(t) = 60t - 240$

For critical numbers,

(i) $C'(t) = 60t - 240 = 0$
$$60t = 240$$
$$t = 4$$

(ii) $C'(t)$ is defined $\forall$ t values in domain.

Check values:

$C(0) = 30(0)^2 - 240(0) + 500 = 500$
$C(4) = 30(4)^2 - 240(4) + 500 = 20$
$C(7) = 30(7)^2 - 240(7) + 500 = 290$

$\therefore$ The lowest concentration during the first week is 20 bacteria/cm^3 which occurs 4 days after treatment.

7. a) $E(v) = \dfrac{1600v}{v^2 + 6400}$ where E is the fuel efficiency, in L per 100 km and v is the speed (km/h) $0 \leq v \leq 100$

$$E'(v) = \frac{1600(v^2+6400) - 1600v(2v)}{(v^2+6400)^2}$$
$$= \frac{1600v^2 + 10\,240\,000 - 3200v^2}{(v^2+6400)^2}$$
$$= \frac{10\,240\,000 - 1600v^2}{(v^2+6400)^2}$$

For critical numbers,

(i) $E'(v) = 0 \Rightarrow 10\,240\,000 - 1600v^2 = 0$
$$v^2 = 6400$$
$$v = 80 \quad (v \geq 0)$$

(ii) $E'(v)$ is defined $\forall$ v in the domain

$\Longrightarrow$

Check values:

$$E(0) = \frac{1600(0)}{(0)^2+6400} = 0$$

$$E(80) = \frac{1600(80)}{(80)^2+6400} = 10$$

∴ The speed limit that maximizes fuel efficiency is 80 km/h.

$$E(100) = \frac{1600(100)}{(100)^2+6400} \doteq 9.76$$

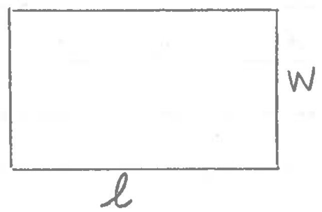
b) If the speed limit is 50 km/h then $0 \leq v \leq 50$ so the critical number at $x=80$ is not part of the domain.

∴ max fuel efficiency is at $v=50$:

$$E(50) = \frac{1600(50)}{(50)^2+6400} \doteq 8.99 \quad L/100\,km.$$

Pg 206 #1–5

1.



Let ℓ be the length of the rectangle and let w be the width. (in cm)

max $A = \ell \cdot w$ given $P = 2(\ell + w) = 100$

$\ell + w = 50$

$\ell = 50 - w$

sub $\ell = 50 - w$

$A = (50 - w)w$

$A = 50w - w^2 \quad (0 \le w \le 50)$

$A' = 50 - 2w$

For critical numbers,

(i) $A' = 50 - 2w = 0$

$50 = 2w$

$w = 25$

(ii) A' is defined $\forall$ values of w in domain

Check: $A(0) = 50(0) - (0)^2 = 0$

$A(25) = 50(25) - (25)^2 = 625$

$A(50) = 50(50) - (50)^2 = 0$

Find ℓ:

$\ell = 50 - (25)$

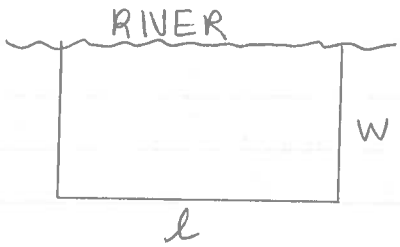
$= 25$ cm

$\therefore$ The largest area is $625\ cm^2$ when the length is 25 cm and the width is 25 cm

2. Given a rectangle with perimeter N units, the maximum area will be obtained when the rectangle has length $= \frac{1}{4}N$ and width $= \frac{1}{4}N$

Hence, the max. area will be $\frac{1}{16}N^2$.

3.



Let ℓ be the length of the field and let w be the width.

max $A = \ell \cdot w$ given $\ell + 2w = 600$

$\ell = 600 - 2w$

sub $\ell = 600 - 2w$

$$A = (600 - 2w)w$$
$$= 600w - 2w^2 \quad (0 \leq w \leq 300)$$

$$A' = 600 - 4w$$

For critical numbers,

(i) $A' = 600 - 4w = 0$

$600 = 4w$

$w = 150$

(ii) A' is defined $\forall$ values of w in the domain.

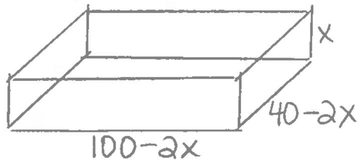
Check: $A(0) = 600(0) - 2(0)^2 = 0$

$A(150) = 600(150) - 2(150)^2 = 45000$

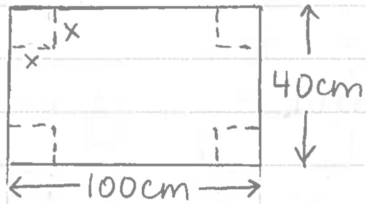
$A(300) = 600(300) - 2(300)^2 = 0$

Find ℓ: $\ell = 600 - 2(150) = 300$

$\therefore$ The maximum area of the field is $45\,000 \text{ m}^2$ when the length is 300 m and the width is 150 m.



4.



Let x be the length of the side of the square cut from each corner.

$$\max \quad V = x(40-2x)(100-2x) \qquad 0 \leq x \leq 20$$
$$= x(4000-80x-200x+4x^2)$$
$$= x(4000-280x+4x^2)$$
$$= 4000x-280x^2+4x^3$$

$$V' = 4000-560x+12x^2$$

For critical numbers,

(i) $V' = 4000-560x+12x^2 = 0$

$4(3x^2-140x+1000) = 0$

(ii) V' is defined $\forall$ values of x in the domain

$$x = \frac{140 \pm \sqrt{(-140)^2-4(3)(1000)}}{2(3)}$$
$$= \frac{140 \pm \sqrt{7600}}{6}$$

$x = 37.86$ OR $x = 8.80$

inadmissible (not in domain)

$l = 100-2(8.8) = 82.4$

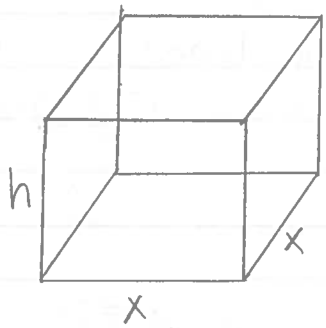
$w = 40-2(8.8) = 22.4$

Check: $V(0) = 4000(0)-280(0)^2+4(0)^3 = 0$

$V(8.8) = 4000(8.8)-280(8.8)^2+4(8.8)^3 \doteq 16243 \text{ cm}^3$

$V(20) = 4000(20)-280(20)^2+4(20)^3 = 0$

$\therefore$ The maximum volume is 16243 cm^3 when the length is 82.4 cm, the width is 22.4 cm and the height is 8.8 cm.

5. 

Let x be the length and width of the square base.
Let h be the height

Given $V = x^2 h = 1000\text{ cm}^3$

$$h = \frac{1000}{x^2}$$

min. $SA = 2x^2 + 4xh$

sub. $h = \frac{1000}{x^2}$

$$SA = 2x^2 + 4x\left(\frac{1000}{x^2}\right)$$

$$= 2x^2 + \frac{4000}{x} \qquad 0 \le x \le ?$$

$$A' = 4x - \frac{4000}{x^2}$$

For critical numbers,

(i) $A' = 4x - \frac{4000}{x^2} = 0$

$$4x = \frac{4000}{x^2}$$

$$4x^3 = 4000$$

$$x^3 = 1000$$

$$x = 10$$

(ii) A' is undefined when $x = 0$

Check using 2nd derivative:

$$A'' = 4 + \frac{8000}{x^3}$$

When $x = 10$,

$$A''(10) = 4 + \frac{8000}{(10)^3} > 0$$

$\therefore$ a min occurs at $x = 10$

Find $h = \frac{1000}{(10)^2} = 10$

$$A = 2(10)^2 + 4(10)(10)$$

$$= 600\text{ cm}^2$$

$\therefore$ The minimum surface area is 600cm^2 when the dimensions are $10\text{ cm} \times 10\text{ cm} \times 10\text{ cm}$.