Graphing Transformations of Trig Functions – A Summary

Given: $y = a\sin[k(\theta - p)] + q$ or $y = a\cos[k(\theta - p)] + q$

Vertical Stretch/Compression/Reflection: * $|a|$ = amplitude

- If $|a| > 1$ then the graph is vertically stretched by factor "a".
- If $0 < |a| < 1$ then the graph is compressed vertically by factor "a".
- if $a < 0$ then the graph is reflected in the x-axis

Horizontal Stretch/Compression/Reflection: * $\text{period} = \frac{360}{|k|}$ Note: k is the number of cycles between 0° and 360°

- If $k > 1$ then the graph is compressed horizontally by factor k.
- If $0 < k < 1$ then the graph is stretched horizontally by factor k.
- If $k < 0$ then the graph is reflected in the y axis.

Vertical Translation:

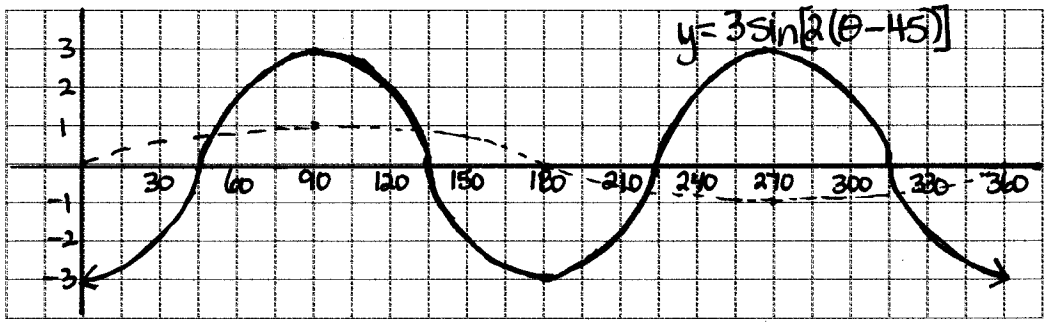
- If $q > 0$, the graph shifts up "q" units
- If $q < 0$, the graph shifts down "q" units

Horizontal Translation (Phase Shift):

- If $p > 0$, the graph shifts right p units
- If $p < 0$, the graph shifts left p units

Eg. 1 Sketch one cycle of $y = 3\sin 2(\theta - 45°)$. State the *domain*, *range* and *phase shift*.

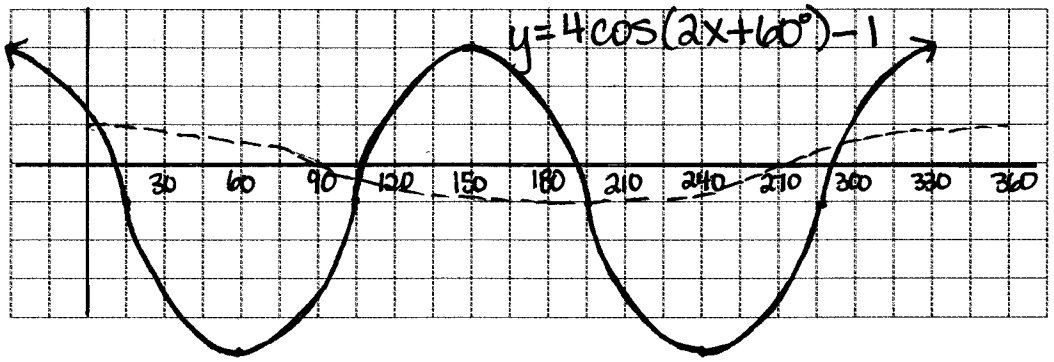
Steps: -vertical stretch factor 3 $\Rightarrow$ amplitude = 3
-horiz. comp factor 2 $\Rightarrow$ period $= \frac{360}{2} = 180°$
-shift right 45°



$D = \{x \in \mathbb{R}\}$
$R = \{y \in \mathbb{R} \mid -3 \le y \le 3\}$
phase shift:
right 45°

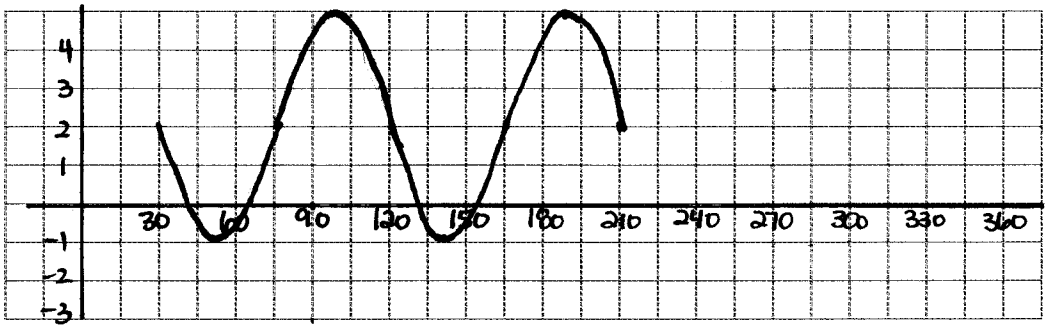
Eg. 2 Sketch $y = 4\cos(2x + 60°) - 1$ on the interval $0° \le x \le 360°$.

Steps: $y = 4\cos[2(x + 30°)] - 1$
-vert. stretch factor 4
-horiz. comp. factor 2
-shift left 30°
-shift down 1



Eg. 3 Sketch one cycle of $y = -3\sin[4(x - 30°)] + 2$.
State the amplitude, period, vertical translation and phase shift.

Steps: -reflection in x axis
-vert. stretch factor 3
-horiz. comp factor 4
-shift right 30°
-shift up 2



amp = 3
period $= \frac{360}{4} = 90$
vert. shift:
up 2
phase shift:
right 30°