# Evaluating Limits Algebraically

① Try substitution first

eg. $\lim_{x\to 3} \sqrt{\frac{6+x}{4-x}}$

$= \sqrt{\frac{6+3}{4-3}} = \sqrt{9} = 3$

② If substitution yields $\frac{0}{0}$ then rewrite the function by doing one of the following

- factor
- rationalize
- common denominator
- substitution

*see eg. from yesterday's lesson

③ If substitution yields $\frac{\#}{0}$ then evaluate the corresponding one-sided limits to determine if two-sided limit exists.

eg. $\lim_{x\to 5} \frac{x}{(x-5)^2}$

*note: substitution yields $\frac{5}{0}$

$\lim_{x\to 5^-} \frac{x}{(x-5)^2}$

*sub. a number slightly less than 5

$\doteq \frac{5}{(-sm)^2}$

$= +\infty$

$\lim_{x\to 5^+} \frac{x}{(x-5)^2}$

sub. a number slightly more than 5

$\doteq \frac{5}{(+sm)^2}$

$= +\infty$

## ④ Limits as $x \to \pm\infty$

### (a) Rational Functions

*Divide each term by the highest power of x in the denominator.

eg. $\lim\limits_{x\to\infty} \dfrac{4x^2 - x + 2}{1 + 5x + 6x^2}$

$= \lim\limits_{x\to\infty} \dfrac{4 - 1/x + 2/x^2}{1/x^2 + 5/x + 6}$

Evaluate using limit properties

$= \dfrac{4}{6} = \dfrac{2}{3}$

### (b) Special Cases

- try substituting a pos/neg "large" number, and generalize to get limit

1) $\lim\limits_{x\to\infty} x^3 - x^2$

$= \lim\limits_{x\to\infty} x^2(x-1) = \infty$

*If possible, factor first then sub. into each factor

as $x \to \infty$, $x^2 \to \infty$, $x - 1 \to \infty$ $\therefore f(x) \to \infty$

2) $\lim\limits_{x\to\infty} 2^x = \infty$

$2^\infty = \infty$

3) $\lim\limits_{x\to-\infty} 2^x = 0$

$2^{-\infty} = \dfrac{1}{2^\infty} = 0$

4) $\lim\limits_{x\to\infty} \left(\dfrac{2}{3}\right)^x$

$= \lim\limits_{x\to\infty} \dfrac{2^x}{3^x}$

$= 0$

As $x \to \infty$
$2^x \to \infty$
$3^x \to \infty$
however $3^x$ is much bigger than $2^x$ so
$\dfrac{2^x}{3^x} \to 0$

5) $\lim\limits_{x\to\infty} \left(\dfrac{3}{2}\right)^x = \infty$

6) $\lim\limits_{x\to\infty} 10^{\frac{1}{x}}$

$= 1$

as $x \to \infty$
$\frac{1}{x} \to 0$