More Business & Economics Optimization - Solutions

1. $C(x) = 280\,000 + 12.5x + 0.07x^2$

a) $c(x) = \dfrac{280\,000 + 12.5x + 0.07x^2}{x}$

$= \dfrac{280\,000}{x} + 12.5 + 0.07x$ (average cost function)

When $x = 1000$,

$c(1000) = \dfrac{280\,000}{1000} + 12.5 + 0.07(1000)$

$= \$362.50/\text{item}$

For marginal cost, $C'(x) = 12.5 + 0.14x$

$C'(1000) = 12.5 + 0.14(1000)$

$= \$152.50/\text{item}$

b) For minimum average cost,

$c'(x) = \dfrac{-280\,000}{x^2} + 0.07$

Solve $c'(x) = 0$:

$\dfrac{-280\,000}{x^2} + 0.07 = 0$

$0.07 = \dfrac{280\,000}{x^2}$

$0.07x^2 = 280\,000$

$x^2 = 4\,000\,000$

$x = 2000$

$\therefore$ A production level of 2000 items will minimize average cost.

c) $c(2000) = \dfrac{280\,000}{(2000)} + 12.5 + 0.07(2000)$

$= \$292.50/\text{item}$

$\therefore$ The min. average cost is $\$292.50$/item.

2. $P(x) = R(x) - C(x)$

$$= (0.68x - 0.00001x^2) - (48000 + 0.28x + 0.00001x^2)$$

$$= 0.4x - 0.00002x^2 - 48\,000$$

$P'(x) = 0.4 - 0.00004x$

For maximum profit, $P'(x) = 0$

$$0.4 - 0.00004x = 0$$

$$0.4 = 0.00004x$$

$$x = 10\,000$$

$\therefore$ They should sell 10 000 cans of soup to maximize profits.

3. $R(x) = x \cdot p(x)$

$$= x \left[\frac{30\,000 - x}{10000}\right]$$

$$= 3x - 0.0001x^2$$

$P(x) = R(x) - C(x)$

$$= (3x - 0.0001x^2) - (6000 + 0.8x)$$

$$= 2.2x - 0.0001x^2 - 6000$$

$P'(x) = 2.2 - 0.0002x$

For maximum profit, $P'(x) = 0$

$$2.2 - 0.0002x = 0$$

$$2.2 = 0.0002x$$

$$x = 11000$$

$\therefore$ They should sell 11 000 subs to maximize profit.

4. a) Let x be the number of tickets sold.
Let p be the price per ticket.

Given $(x_1, p_1) = (27000, 30)$ and $(x_2, p_2) = (33000, 25)$

Assuming the demand function is linear,

$$m = \frac{30-25}{27000-33000}$$

$$= \frac{5}{-6000} = \frac{1}{-1200}$$

The demand function is given by:

$$\frac{1}{-1200} = \frac{p-30}{x-27000}$$

$$-1200p + 36000 = x - 27000$$

$$-1200p = x - 63000$$

$$p = \frac{-1}{1200}x + 52.5$$

$\therefore$ The demand (price) function is $p(x) = \frac{-1}{1200}x + 52.5$

b) $R(x) = x \cdot p(x)$

$$= x\left(\frac{-1}{1200}x + 52.5\right)$$

$$= \frac{-1}{1200}x^2 + 52.5x$$

$$R'(x) = \frac{-1}{600}x + 52.5$$

Solve $R'(x) = 0$:

$$52.5 = \frac{1}{600}x$$

$$x = 31500$$

Sub $x = 31500$ in to $p(x)$

$$p(31500) = \frac{-1}{1200}(31500) + 52.5$$

$$= \$26.25$$

If the owners set the ticket price at \$26.25, they will sell 31500 tickets to maximize revenue.

5. Let x be the number of units rented.
Let p be the monthly price per unit

Given $(x_1, p_1) = (120, 400)$ and $(x_2, p_2) = (119, 410)$

Assuming the demand function is linear,

$$m = \frac{400-410}{120-119} = -10$$

The demand (price) function is given by:

$$-10 = \frac{p-400}{x-120}$$

$$p - 400 = -10x + 1200$$

$$p = -10x + 1600$$

Revenue function: $R(x) = x \cdot p(x)$

$$= x(-10x + 1600)$$

$$= -10x^2 + 1600x$$

$$R'(x) = -20x + 1600$$

For maximum revenue, $R'(x) = 0$

$$-20x + 1600 = 0$$

$$1600 = 20x$$

$$x = 80$$

Sub $x = 80$ in to $p(x)$:

$$p(80) = -10(80) + 1600$$

$$= \$800$$

$\therefore$ If she charges \$800/unit, she will rent 80 units to maximize revenue.