

## Derivative Practice - Solutions

- ①  $y = 5 - x^3$   
 $\frac{dy}{dx} = -3x^2$
- ②  $y = \sqrt[3]{x^2} = x^{2/3}$   
 $\frac{dy}{dx} = \frac{2}{3} x^{-1/3}$   
 $= \frac{2}{3\sqrt[3]{x}}$
- ③  $f(x) = ax^2 + bx + c$   
 $f'(x) = 2ax + b$
- ④  $s(t) = 4t - \frac{4}{\sqrt{t}}$   
 $= 4t - 4t^{-1/2}$   
 $s'(t) = 4 + 2t^{-3/2}$   
 $= 4 + \frac{2}{t^{3/2}}$
- ⑤  $f(x) = \frac{2}{\sqrt[3]{x}} + \frac{3}{x^2}$   
 $= 2x^{-1/3} + 3x^{-2}$   
 $f'(x) = -\frac{2}{3} x^{-4/3} - 6x^{-3}$   
 $= -\frac{2}{3x^{4/3}} - \frac{6}{x^3}$
- ⑥  $y = \frac{\sqrt{x}}{2} + \frac{x}{\sqrt{2}}$   
 $= \frac{1}{2} x^{1/2} + \frac{1}{\sqrt{2}} x$   
 $\frac{dy}{dx} = \frac{1}{4} x^{-1/2} + \frac{1}{\sqrt{2}}$   
 $= \frac{1}{4\sqrt{x}} + \frac{1}{\sqrt{2}}$
- ⑦  $f(x) = (3x^2 + 2)(x^3 - 1)$   
 $f'(x) = 6x(x^3 - 1) + (3x^2 + 2)(3x^2)$   
 $= 6x^4 - 6x + 9x^4 + 6x^2$   
 $= 15x^4 + 6x^2 - 6x$   
 $= 3x(5x^3 + 2x - 2)$
- ⑧  $y = \frac{x^2 - 4}{3x + 2}$   
 $y' = \frac{2x(3x + 2) - (x^2 - 4)(3)}{(3x + 2)^2}$   
 $= \frac{6x^2 + 4x - 3x^2 + 12}{(3x + 2)^2}$   
 $= \frac{3x^2 + 4x + 12}{(3x + 2)^2}$
- ⑨  $f(x) = \frac{x^{3/2} - x^{1/2} + 3}{x^{1/2}}$   
 $= x - 1 + 3x^{-1/2}$   
 $f'(x) = 1 - \frac{3}{2} x^{-3/2}$   
 $= 1 - \frac{3}{2x^{3/2}}$

$$\textcircled{10} \quad y = (1-x)(2\sqrt{x}+1)$$

$$\begin{aligned} \frac{dy}{dx} &= (-1)(2\sqrt{x}+1) + (1-x)(x^{-1/2}) \\ &= -2x^{1/2} - 1 + x^{-1/2} - x^{1/2} \\ &= -3x^{1/2} - 1 + x^{-1/2} \\ &= -3\sqrt{x} + \frac{1}{\sqrt{x}} - 1 \end{aligned}$$

$$\textcircled{12} \quad y = 4\sqrt{9-2x^3}$$

$$= 4(9-2x^3)^{1/2}$$

$$\begin{aligned} \frac{dy}{dx} &= 2(9-2x^3)^{-1/2}(-6x^2) \\ &= \frac{-12x^2}{\sqrt{9-2x^3}} \end{aligned}$$

$$\textcircled{14} \quad y = \left(\sqrt[3]{x} - \frac{1}{\sqrt[3]{x}}\right)^3$$

$$= (x^{1/3} - x^{-1/3})^3$$

$$\begin{aligned} y' &= 3(x^{1/3} - x^{-1/3})^2 \left(\frac{1}{3}x^{-2/3} + \frac{1}{3}x^{-4/3}\right) \\ &= 3 \cdot \frac{1}{3} (x^{1/3} - x^{-1/3})^2 (x^{-2/3} + x^{-4/3}) \\ &= \frac{1}{x^{4/3}} (x^{1/3} - x^{-1/3})^2 (x^{2/3} + 1) \end{aligned}$$

$$\textcircled{16} \quad f(x) = x^2(4x-5)^6$$

$$\begin{aligned} f'(x) &= 2x(4x-5)^6 + x^2[6(4x-5)^5(4)] \\ &= 2x(4x-5)^5 [(4x-5) + 12x] \\ &= 2x(4x-5)^5 (16x-5) \end{aligned}$$

$$\textcircled{11} \quad h(x) = (x^3+3x^2-2)^5$$

$$\begin{aligned} h'(x) &= 5(x^3+3x^2-2)^4(3x^2+6x) \\ &= 15x(x+2)(x^3+3x^2-2)^4 \end{aligned}$$

$$\textcircled{13} \quad y = \frac{4}{(5x^2+1)}$$

$$= 4(5x^2+1)^{-1}$$

$$\begin{aligned} \frac{dy}{dx} &= -4(5x^2+1)^{-2}(10x) \\ &= \frac{-40x}{(5x^2+1)^2} \end{aligned}$$

$$\textcircled{15} \quad s(t) = \frac{1}{\sqrt{b+2t}}$$

$$= (b+2t)^{-1/2}$$

$$\begin{aligned} s'(t) &= -\frac{1}{2}(b+2t)^{-3/2}(2) \\ &= \frac{-1}{(b+2t)^{3/2}} \end{aligned}$$

$$\textcircled{7} \quad g(x) = [x^2(4x-5)]^6$$

$$\begin{aligned} g'(x) &= 6[x^2(4x-5)]^5 [2x(4x-5) + x^2(4)] \\ &= 6[x^2(4x-5)]^5 [8x^2 - 10x + 4x^2] \\ &= 6[x^2(4x-5)]^5 (12x^2 - 10x) \\ &= 12x(6x-5)[x^2(4x-5)]^5 \\ &= 12x^7(6x-5)(4x-5)^5 \end{aligned}$$

$$\textcircled{8} \quad y = \frac{3x+2}{(x^3-5)^6}$$

$$\begin{aligned} y' &= \frac{3(x^3-5)^6 - (3x+2)[6(x^3-5)^5(3x^2)]}{(x^3-5)^{12}} \\ &= \frac{3(x^3-5)^5 [(x^3-5) - (3x+2)6x^2]}{(x^3-5)^{12}} \\ &= \frac{3[x^3-5-18x^3-12x^2]}{(x^3-5)^7} \\ &= \frac{-3(17x^3+12x^2+5)}{(x^3-5)^7} \end{aligned}$$

$$\textcircled{9} \quad y = \left(\frac{3x+2}{x^3-5}\right)^6$$

$$\begin{aligned} y' &= 6\left(\frac{3x+2}{x^3-5}\right)^5 \left[\frac{3(x^3-5) - (3x+2)(3x^2)}{(x^3-5)^2}\right] \\ &= 6\left(\frac{3x+2}{x^3-5}\right)^5 \left[\frac{3x^3-15-9x^3-6x^2}{(x^3-5)^2}\right] \\ &= \frac{6(3x+2)^5(-6x^3-6x^2-15)}{(x^3-5)^7} \\ &= \frac{-18(3x+2)^5(2x^3+2x^2+5)}{(x^3-5)^7} \end{aligned}$$

$$\textcircled{20} \quad y = (5x+3)^2 \sqrt{x^3-1}$$

$$\begin{aligned} \frac{dy}{dx} &= [2(5x+3)(5)](x^3-1)^{1/2} + (5x+3)^2 \left[ \frac{1}{2}(x^3-1)^{-1/2}(3x^2) \right] \\ &= 10(5x+3)(x^3-1)^{1/2} + \frac{3}{2}x^2(5x+3)^2(x^3-1)^{-1/2} \\ &= \frac{1}{2}(5x+3)(x^3-1)^{-1/2} [20(x^3-1) + 3x^2(5x+3)] \\ &= \frac{5x+3}{2\sqrt{x^3-1}} (20x^3-20+15x^3+9x^2) \\ &= \frac{(5x+3)(35x^3+9x^2-20)}{2\sqrt{x^3-1}} \end{aligned}$$

$$\textcircled{21} \quad y = \frac{4x-7}{\sqrt{2x+3}}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{4(2x+3)^{1/2} - (4x-7)\left[\frac{1}{2}(2x+3)^{-1/2}(2)\right]}{2x+3} \\ &= \frac{(2x+3)^{-1/2} [4(2x+3) - (4x-7)]}{2x+3} \\ &= \frac{4x+19}{(2x+3)^{3/2}} \end{aligned}$$

$$\textcircled{22} \quad f(x) = \sqrt{(2x-5)(x^2+1)^3} = [(2x-5)(x^2+1)^3]^{1/2}$$

$$\begin{aligned} f'(x) &= \frac{1}{2} [(2x-5)(x^2+1)^3]^{-1/2} [2(x^2+1)^3 + (2x-5)[3(x^2+1)^2(2x)]] \\ &= \frac{1}{2} [(2x-5)(x^2+1)^3]^{-1/2} [2(x^2+1)^3 + 6x(2x-5)(x^2+1)^2] \\ &= \frac{1}{2} [(2x-5)(x^2+1)^3]^{-1/2} [2(x^2+1)^2 [(x^2+1) + 3x(2x-5)]] \\ &= \frac{2(x^2+1)^2(7x^2-15x+1)}{2\sqrt{(2x-5)(x^2+1)^3}} \\ &= \frac{(x^2+1)^2(7x^2-15x+1)}{\sqrt{(2x-5)(x^2+1)^3}} = \frac{\sqrt{x^2+1}(7x^2-15x+1)}{\sqrt{2x-5}} \end{aligned}$$

$$\textcircled{23} \quad y = 2x^3 \sqrt{x-1} (4x+5)^6$$

$$y' = 6x^2(\sqrt{x-1})(4x+5)^6 + 2x^3 \left[ \frac{1}{2}(x-1)^{-1/2} (4x+5)^6 + (x-1)^{1/2} (6)(4x+5)^5 (4) \right]$$

$$\textcircled{24} \quad y = \frac{\sqrt[3]{4x-3}}{(x^3+4)^4}$$

$$\frac{dy}{dx} = \frac{\left[ \frac{1}{3}(4x-3)^{-2/3} (4) \right] (x^3+4)^4 - (4x-3)^{1/3} \left[ 4(x^3+4)^3 (3x^2) \right]}{(x^3+4)^8}$$

$$\textcircled{25} \quad y = \left[ \frac{2x^3+x^2-9}{x^4 \sqrt{3x+1}} \right]^5$$

$$\frac{dy}{dx} = 5 \left[ \frac{2x^3+x^2-9}{x^4 \sqrt{3x+1}} \right]^4 \left[ \frac{(6x^2+2x)(x^4 \sqrt{3x+1}) - (2x^3+x^2-9) \left[ 4x^3 (3x+1)^{1/2} + x^4 \left( \frac{1}{2} (3x+1)^{-1/2} \right) \right]}{x^8 (3x+1)} \right]$$