Derivative Practice - Combining the Rules

When more than one derivative rule is needed to differentiate a function, determine the "major" rule as follows:

1) consider how you would evaluate $f(a)$ for a given value of $x=a$, following order of operations
2) the last operation will indicate the first derivative rule, or "major" rule.

Eg. Find the derivative and simplify completely.

(a) $f(x)=(3x^2+5)^4\sqrt{1-2x}$

$$f'(x)=[4(3x^2+5)^3(6x)](1-2x)^{1/2}+(3x^2+5)^4\left[\tfrac{1}{2}(1-2x)^{-1/2}(-2)\right]$$

$$=24x(3x^2+5)^3(1-2x)^{1/2}-(3x^2+5)^4(1-2x)^{-1/2}$$

$$=\underbrace{(3x^2+5)^3(1-2x)^{-1/2}}_{\text{common factor}}\left[24x(1-2x)-(3x^2+5)\right]$$

$$=\frac{(3x^2+5)^3\,[24x-48x^2-3x^2-5]}{(1-2x)^{1/2}}$$

$$=\frac{(3x^2+5)^3(-51x^2+24x-5)}{(1-2x)^{1/2}}$$

*Note: When evaluating $f(a)$, the last operation would be multiplication $\Rightarrow\therefore$ the "major" derivative rule is the product rule

(b) $y = \dfrac{\sqrt{4x^3-1}}{(3-2x^4)^2}$

$$\frac{dy}{dx} = \frac{\left[\frac{1}{2}(4x^3-1)^{-1/2}(12x^2)\right](3-2x^4)^2 - (4x^3-1)^{1/2}\left[2(3-2x^4)(-8x^3)\right]}{(3-2x^4)^4}$$

$$= \frac{6x^2(4x^3-1)^{-1/2}(3-2x^4)^2 + 16x^3(3-2x^4)(4x^3-1)^{1/2}}{(3-2x^4)^4}$$

$$= \frac{2x^2(4x^3-1)^{-1/2}(3-2x^4)\left[3(3-2x^4)+8x(4x^3-1)\right]}{(3-2x^4)^4}$$

$$= \frac{2x^2(4x^3-1)^{-1/2}(3-2x^4)(9-6x^4+32x^4-8x)}{(3-2x^4)^4}$$

$$= \frac{2x^2(26x^4-8x+9)}{(4x^3-1)^{1/2}(3-2x^4)^4}$$

Guidelines for Simplifying Derivatives

① Do any "obvious" simplification ie. easy multiplying, collect like terms, etc.

② Factor, if possible.

③ Rewrite terms so all exponents are positive.

④ Combine terms in each factor by getting a common denominator, if necessary.

NOTE: You do <u>not</u> usually need to expand powers of polynomials such as $(2x+3)^4$.

HW: Worksheet "Derivative Practice"