Chain Rule – The Proof

$$\boxed{\text{If } \quad f(x)=[g(x)]^n, \text{ then } \; f'(x)=n[g(x)]^{n-1}\cdot g'(x)}$$

Proof: (assuming knowledge of product rule)

$$\begin{aligned} f(x) &= [g(x)]^n \\ &= \underbrace{g(x)\cdot g(x)\cdot g(x)\cdot \ldots \cdot g(x)}_{n \text{ terms}} \end{aligned}$$

$$\left.\begin{aligned} f'(x) = g'(x)&\Big[\cdot\underbrace{g(x)\cdot g(x)\cdot\ldots\cdot g(x)}_{n-1\text{ terms}}\Big] \\ &+ g(x)\Big[\cdot g'(x)\Big[g(x)\cdot g(x)\cdot\ldots\cdot g(x) \\ &+ g(x)\cdot g(x)\cdot g'(x)\cdot g(x)\ldots\cdot g(x) \\ &+ \\ &\;\vdots \\ &+ g(x)\cdot g(x)\cdot g(x)\cdot\ldots\cdot g'(x)\end{aligned}\right\} n \text{ times}$$

$$\therefore f'(x) = n[g(x)]^{n-1}\cdot g'(x)$$