Average Rate of Change

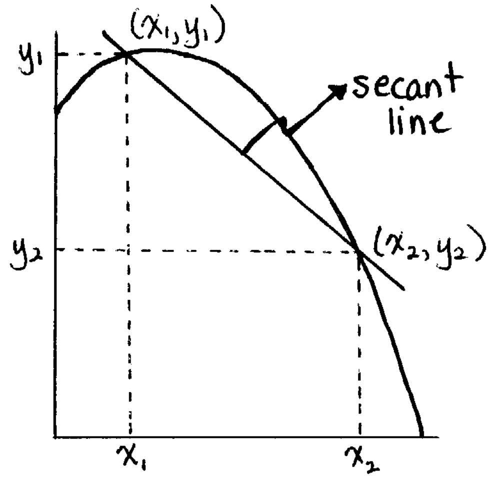
The **average rate of change** is the change in the quantity of the dependent variable (Δy) divided by the change in the quantity of the independent variable (Δx) over a defined interval. Algebraically this can be represented as follows:

Average rate of change (AROC) = change in y / change in x

$$= \frac{\Delta y}{\Delta x} \quad \text{OR} \quad \frac{\Delta f(x)}{\Delta x}$$

$$\frac{\Delta f(x)}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$



for the interval $x_1 \leq x \leq x_2$.

Graphically, the average rate of change for any function y = f(x) over the interval $x_1 \leq x \leq x_2$ is equivalent to the slope of the secant line (a line that passes through two points on the graph of a relation) passing through two points (x_1, y_1) and (x_2, y_2).

A **positive** rate of change indicates that the dependent variable is increasing and the slope of the secant line is positive.

A **negative** rate of change is indicates that the dependant variable is decreasing and the slope of the secant line is negative.

All linear relationships gave a constant rate of change.

Nonlinear relationships do not have a constant rate of change. Average rate of change calculations over different intervals give different results.

Example 1: Calculate the average rate of change for the function $f(x) = 2x^2 + 3x - 1$ over the interval $1 \leq x \leq 3$.

$$\frac{\Delta f(x)}{\Delta x} = \frac{f(3) - f(1)}{3-1}$$

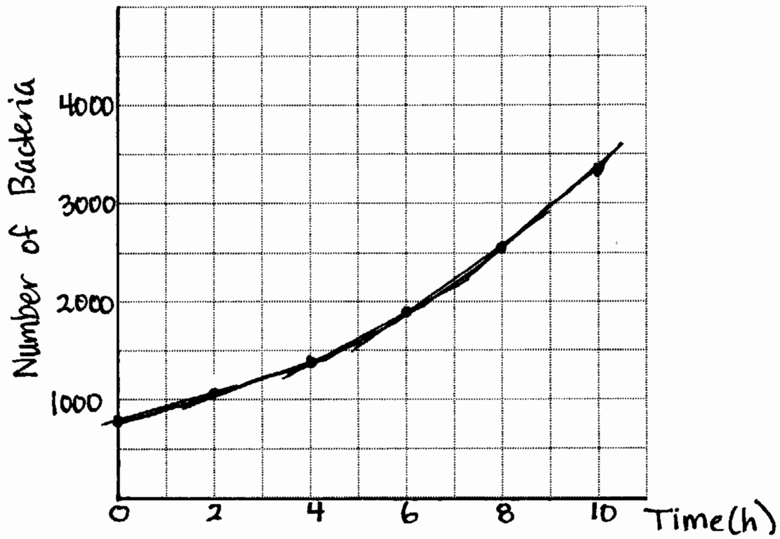
$$= \frac{[2(3)^2 + 3(3) - 1] - [2(1)^2 + 3(1) - 1]}{3-1}$$

$$= \frac{(26) - (4)}{2}$$

$$= \frac{22}{2} = 11$$

Example 2: The following table represents the growth of a bacteria population over a 10 hour period.

Time (h)	Number of Bacteria
0	850
2	1122
4	1481
6	1954
8	2577
10	3400



a) Plot the points on the above grid. Draw a secant line that passes through the endpoints for each 2-hour interval.

b) Find the average rate of change of the number of bacterial in each two hour interval.

$$\underset{(t=0 \to t=2)}{AROC} = \frac{\Delta b}{\Delta t} = \frac{1122-850}{2-0} = \frac{272}{2} = 136 \text{ bacteria/hr}$$

$$\underset{(t=2 \to t=4)}{AROC} = \frac{\Delta b}{\Delta t} = \frac{1481-1122}{4-2} = \frac{359}{2} = 179.5 \text{ bacteria/hr}$$

$$\underset{(t=4 \to t=6)}{AROC} = \frac{\Delta b}{\Delta t} = \frac{1954-1481}{6-4} = \frac{473}{2} = 236.5 \text{ bacteria/hr}$$

$$\underset{(t=6 \to t=8)}{AROC} = \frac{\Delta b}{\Delta t} = \frac{2577-1954}{8-6} = \frac{623}{2} = 311.5 \text{ bacteria/hr}$$

$$\underset{(t=8 \to t=10)}{AROC} = \frac{\Delta b}{\Delta t} = \frac{3400-2577}{10-8} = \frac{823}{2} = 411.5 \text{ bacteria/hr}$$

c) In which interval did the bacterial population grow fastest?

The bacteria grew fastest from 8 h to 10 h.

d) Determine the average rate of change of the number of bacteria from 6 hours to 10 hours. How does this compare to the average rate of change from 8 hours to 10 hours?

$$\underset{(t=6 \to t=10)}{AROC} = \frac{\Delta b}{\Delta t} = \frac{3400-1954}{10-6} = \frac{1446}{4} = 361.5 \text{ bacteria/hr}$$

The average rate of change from 6 h to 10 h is 361.5 bacteria/hr, which is less than the average rate of change from 8 h to 10 h.