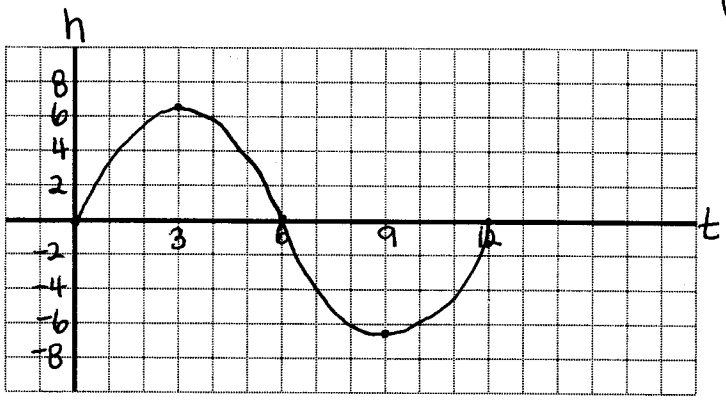
Applications of Trig Functions

Eg. 1 Ocean Tides

The alternating half-daily cycles of the rise and fall of oceans are called tides. Tides in one section of the Bay of Fundy caused the water level to rise 6.5 m above the mean sea level and to drop 6.5 m below. The tide completes one cycle approximately every 12 hours. Draw a graph and find an equation of the height of water (with respect to mean sea level) as a function of time.

Solution:

① Make a chart to show the water level after "t" hours



t	h
0	0
3	6.5
6	0
9	-6.5
12	0

③ Graph looks similar to $y=\sin x$, so use $y = a\sin[k(x-p)]+q$

∵ no phase shift ($p=0$) and no vert. shift ($q=0$)

② Find amplitude & period:

$amp = \frac{|6.5-(-6.5)|}{2}$

$a = 6.5$

From graph, period = 12 h

$\frac{360}{k} = 12$

$\therefore k = 30$

$\therefore y = 6.5\sin 30x$

or

$\boxed{h = 6.5\sin 30t}$

Eg. 2 The water depth in a harbour is 16 m at high tide and 8 m at low tide. Once cycle is completed approximately every 12 hours.

a) Find an equation for the water depth as a function of time, after low tide.
b) Draw a graph of the function for a 24 hour period starting at 12am.
c) State the times at which the water depth was a maximum.
d) Determine the depth after 7 hours.
e) Determine the time when the depth is 11m.

Solution:

① Chart: (start at 8m)

t	h
0	8
3	12
6	16
9	12
12	8

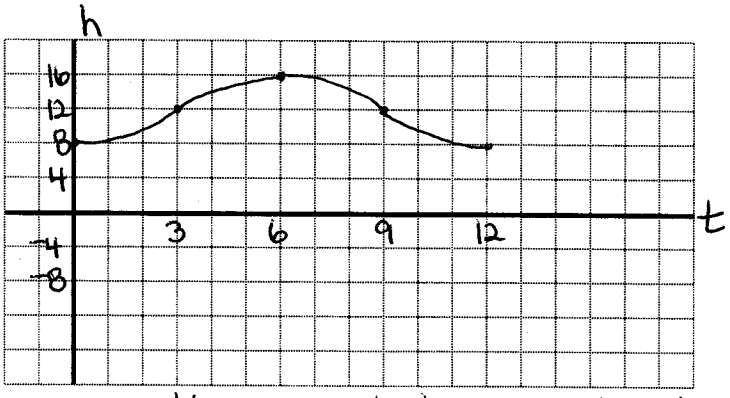
② Amplitude:

$a = \frac{|16-8|}{2} = 4$

Period = 12 h

$\frac{360}{k} = 12$

$k = 30$



③ Notice the graph is a vertical reflection of $y=\cos x$. ∴ use $h=-\cos t$ as base function

∵ no phase shift ($p=0$) and vert shift is $q=12$

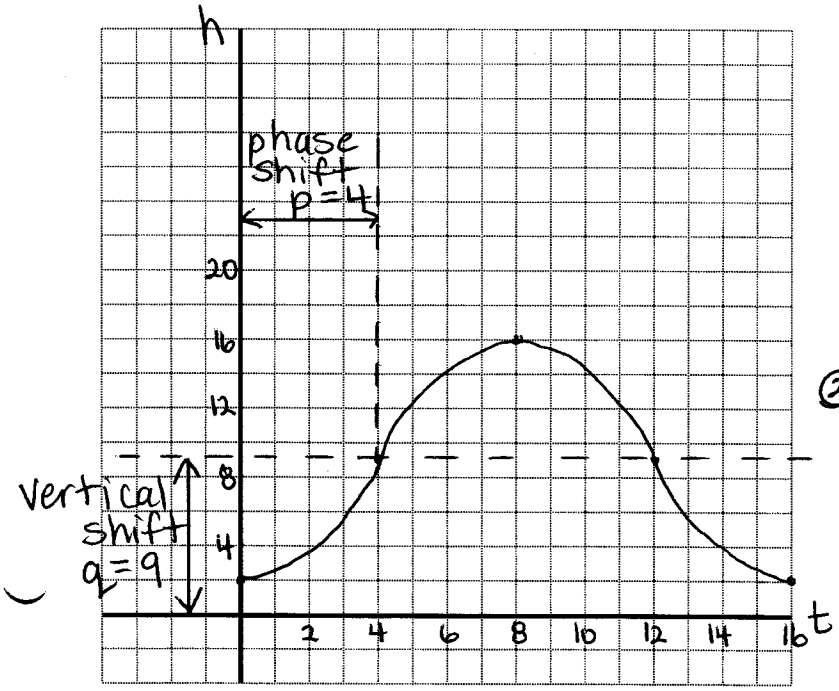
$\therefore \boxed{h = -4\cos 30t + 12}$

Eg. 3 Ferris Wheel

A carnival ferris wheel with a radius of 7m makes one complete revolution every 16 seconds. The bottom of the wheel is 2m above the ground. Draw a graph to show how a person's height above the ground varies with time. Find an equation of the graph.

Solution:

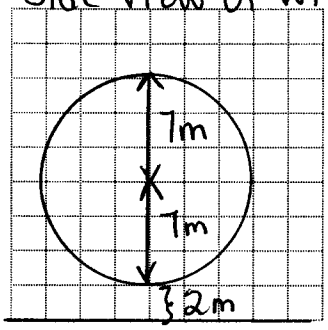
i) Sine Function



① Chart:

t	h
0	2
4	9
8	16
12	9
16	2

Side View of Wheel:



② Amplitude: $a = \frac{|16-2|}{2} = 7$

Period = 16 sec $\Rightarrow \frac{360}{k} = 16$

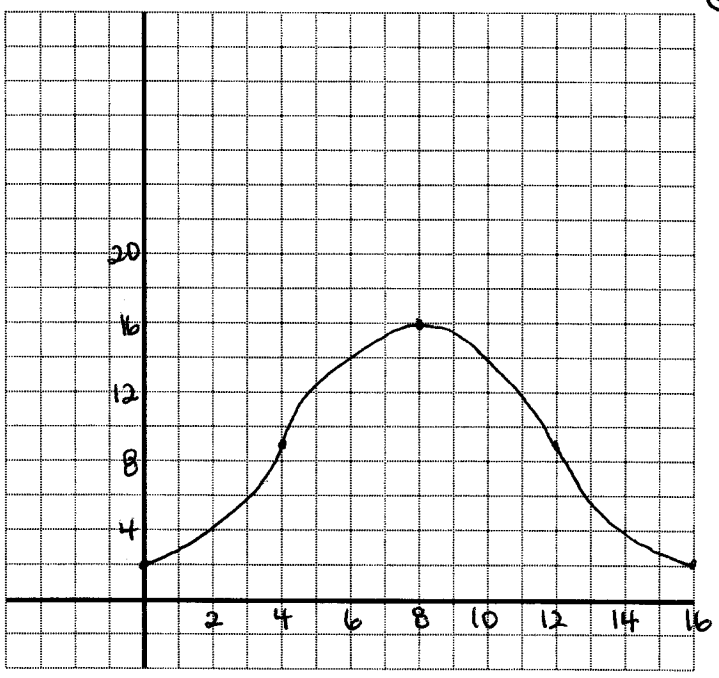
$k = 22.5$

From graph, vertical shift is $q = 9$

③ For sine curve, need phase shift of $p = 4$

$$\therefore h = 7\sin[22.5(t-4)] + 9$$

ii) Cosine Function



④ For cosine curve, no phase shift is needed ($p = 0$), but graph is reflected in x axis ($a < 0$)

$$\therefore h = -7\cos(22.5t) + 9$$